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THE EFFECT OF SOCIAL VALUE MEASUREMENT ON IMPACT INVESTMENT DECISIONS

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Abstract. *Impact investing has great potential to address social problems by tapping into investors' enthusiasm for underpinning activities that 'make a difference to society'. Measurement of social benefits can therefore influence investor choices as perceptions of non-financial returns vary. To understand such choices, financial returns, social and environmental returns (SER) and SER measurement should all be taken into account. There has, however, been little consideration of these aspects in an integrated way. To advance the debate, this paper explores a basic model which extends a standard utility function to SER and draws on insights from theories of knowledge. Indicative results when plausible parameters from research and historical trends are put in indicate (i) a tightness of resources for measurement; (ii) investors becoming less willing to accept lower financial returns when they are less confident about social returns; and (iii) an increased lack of willingness to accept lower financial returns where SER returns relate to intangibles that are harder to measure.*

Keywords: *Social and environmental returns, social value, impact measurement, impact investment*

JEL Classifications*: D81, G11, G14, H23*

Introduction

Key networks such as the G8, World Economic Forum and signatories to the UN Principles for Responsible Investment have signalled support for action by investors to recognise wider social and environmental goals, both in investment decisions and reporting. Impact investing is already a major factor in sectors as varied as micro-finance, social housing, 'clean technology' and water purification (Clearly So, 2011), and the scope for further growth in the sector is huge (O'Donohoe et al, 2010).

Prospects for growth in this sector are enhanced the more that investors and advisers can harness measurement and assessment processes to place a value, implicitly or explicitly, on the wider social and environmental consequences of their choices. Such a theme is widely recognised – both from a corporate perspective (the 'triple bottom line' approach initially advocated in Elkington, 1997), an accounting outlook (the 'Blended Value' approach outlined in Nicholls, 2009), and in social investment (as with the 'Implied Impact' approach of Evenett and Richter, 2012). There has, however, been little conceptual modelling of the interplay as to *how* measurement of impact, level of impact, and financial goals together affect investors' decisions. One important exception, Grabenwarter and Liechtenstein (2011: 54-56), outlines a route to assessing impact within the context of an integrated finance/impact model; however this argues that there is no trade-off between financial return and social impact provided that the investor takes a professional approach. This leaves the inter-linkages of investors' perceptions of financial return, social return and measurement underexplored.

Yet equilibrium can depend greatly on the workings of measurement and information, as is clear from the explorations of multiple equilibria in markets involving asymmetric information initiated by George Akerlof, and the prediction of inefficient financial markets due to the non-zero cost of collecting information on financial assets set out in Grossman and Stiglitz (1980).

A range of practitioners into the processes of impact investing (Reeder et al, 2014) have testified that the measurement of SER is far from costless and far from readily transferable. Managers of impact investing funds frequently spoke of highly constrained budgets for measurement; and highly varying levels of interest among their clients into the social and environmental impacts of their investments. Procedural aspects (such as screening) tend to dominate, with much less attention given to outcomes and attributed changes in outcomes. Other research, such as Paetzold and Busch (2014), highlights other measurement issues, finding a lack of understanding about social investment preferences among fund managers, and a somewhat unfounded perception of additional volatility from social investments amongst clients.

This paper therefore aims to move analysis forward on both conceptual and empirical levels by considering (i) a basic model of investment decisions that integrates financial and non-financial value; (ii) an extension of the model to include choices between investing in assets as opposed to funding better measurement of the non-financial returns from assets; and (iii) implications when broad-brush empirical parameters are deployed.

Model for integrating social and environmental returns into investment decision-making

Current standard models set out to maximise wealth over a series of distinct time periods. A natural extension of that approach is to draw a distinction between wealth (‘w’) and wider wealth (‘ww’), where wider wealth reflects such features as satisfaction with promoting a sustainable environment and supporting a more cohesive community.

The model presented here is based on a choice between two assets:

- An asset with risk-free financial returns (rf); and
- A risky asset that carries both financial returns (r), and social and environmental returns (SER).

The model starts at point t-1, in which the decision is maximise the expected utility of invest what you have ($w_{t-1} = 1$), in volumes of risk free asset $q_{m\ t-1}$ and risky asset $q_{a\ t-1}$ at t-1 to receive back at time t the relevant rates of return relating to those assets, including social and environmental returns (SER_t) that are proportional to the financial rates of return for the risky asset. Then the key variables to follow through on this model are:

- The returns to the risk free asset (i), which is a constant;
- The financial returns to the risky asset (r_t), equating to $\beta_r + \varepsilon_{rt}$ where ε_r is distributed normally with mean 0 and positive variance σ^2_R;
- The extent to which a unit of value of wider benefits for society is weighted by the investor compared to receiving a unit of financial value – expressed by a factor d, where $0 \leq d \leq 1$, so that a level of 0 represents no weight given to such benefits, while a value of 1 implies equal weighting with one’s own financial assets;
- A proportional correlation, α_{SER} between the value of financial assets and the wider social benefits associated with those assets;
- The degree of risk aversion, δ, which is greater than 0.

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The maximisation problem is then:

> (1) Max E_{t-1} [U (ww_t)] through choice of $q_{m\ t-1}$ and $q_{a\ t-1}$ subject to a wealth constraint of no borrowing.

A simple utility function (2) $U(ww_t) = E(ww_t) - \delta / (2\ ww_{t-1}) * Var(ww_{t-1})$ is adopted, using the approximation suggested in Pratt (1964).

The solution (see equation 13, Annex 1) is to adopt a proportion of risky assets in accordance with (3) $p_{a\ t-1}\ \bar{q}_{a\ t-1} / w_{t-1} = \{1+\beta_r\}/\{\delta\ \sigma^2_R\ [1+d\ \alpha_{SER}]\} - (1+i)/\{\delta\ \sigma^2_R\ [1+d\ \alpha_{SER}]^2\}$

Implications for impact investors' choices

Tables 1 and 2 below show the way that investors' choices vary with respect to trade-offs between (a) SER weighting factor d, and expected rate of return on the risky asset, β_r and(b) correlation in returns, α_{SER} , and expected rate of return on the risky asset, β_r.

The tables show the relationship expressed in terms of which parameters keep the proportion of risk-free assets at points between 0% and 100%. In calculating the relationship, the following indicative values were used for variables:

- α_{SER} is assumed equal to 0.40, reflecting a clear but not extremely close correlation between financial and non-financial returns;
- The risk free interest rate is assumed equal to 0%;
- Delta is assumed equal to 0.115, from Bliss and Panigirtzoglu (2004); and
- Volatility in the rate of return of the risky asset is 0.064 (as per the variance in the FTSE small capitalisation index of monthly returns over the period 2003 to 2012).

Table 1 shows the required rate of return under different SER weighting factor assumptions.

Table 1 Rates of return – base case model

SER weighting factor	Required beta (risky assets = 0%)	Required beta (risky assets = 100%)
0%	0.00%	0.74%
5%	-1.96%	-1.21%
10%	-3.85%	-3.08%

In this model, a comparison of the figures in row 1 indicates that a premium of 0.74% is needed for the risky asset to overcome the drawbacks of risk aversion to volatility in returns. The comparison between the figures in row 3 and row 2, and also between row 2 and row 1, indicates the order of magnitude by which those who put a weighting on SER are prepared to accept a lower financial rate of return (this is a separate issue from whether in practice there *is* a lower financial rate of return from social investment, which is an ongoing debate that is beyond the remit of this paper).

Table 2 shows required rates of return for different correlation levels and 5% SER weighting.

Table 2 impact of different correlation levels in base case model

Alpha (SER) correlation factor	Required beta (risky assets = 0%)	Required beta (risky assets = 100%)
0%	0.00%	0.74%
40%	-1.96%	-1.21%
100%	-4.76%	-4.16%

The model implies that with no correlation to SER, the investor makes a decision based on financial return, regardless of their preferences for SER. It also indicates the extent to which those who put a weighting on SER are prepared to accept a lower financial rate of return as correlation rises between the financial return of the asset and SER.

Inclusion of the 'state of knowledge' with respect to SER

In practice, there tends to be much uncertainty on the extent of correlation between SER_t and financial returns. This raises the question of the extent to which perceived social benefits should be discounted in the utility function, in the light of the uncertainty as to how 'true' they are. At least two possible routes can be identified, relating to information theory and evidence theory, which we consider in turn.

Information theory model

The first is to harness findings in information theory deriving from those set out in Weaver and Shannon (1963), reinterpreted for an investment setting in Chen (2007). Under this theory, the state of knowledge is improved when subjective views move closer to objective parameters, and is defined as:

(5) $D(p \parallel q) = \sum_{j=1}^{n} p_j \log(p_j) - \sum_{j=1}^{n} p_j \log(q_j)$, where (p_j) reflects objective parameters, and (q_j) reflects subjective views of those parameters. For our purposes, this index is translated into a discount factor, bounded between 0 and 1, using the simple equation:
(6) sk = {max(D) – D} / max(D), where D is the state of knowledge coefficient, and max(D) is the largest number that it could take in a plausible situation.

Simulating generic states for a given level of knowledge on the social and environmental returns of a project gives the results outlined in table 3 below.

Table 3 Levels of 'state of knowledge' where twenty levels of knowledge are possible

	Probability correct	*State of knowledge coefficient (D)*	*Discount factor (sk)*
Completely unclear	0.05	3.00 = 1 * ln(1) – 1 * ln (1/20)	0%
Initial thinking	0.33	1.10 = 1 * ln(1) – 1 * ln (1/3)	63%
Know if +ve or -ve	0.50	0.69 = 1 * ln(1) – 1 * ln (1/2)	77%
Strength of +ve or -ve	0.67	0.41 = 1 * ln(1) – 1 * ln (2/3)	86%
Detailed knowledge	0.80	0.22 = 1 * ln(1) – 1 * ln (4/5)	93%
Excellent knowledge	0.95	0.05 = 1 * ln(1) – ln (19/20)	98%

When the state of knowledge of the SER achieved is unclear, then the reported level of SER is fully discounted; when the state of knowledge of SER is excellent, the reported level of SER is not discounted.

This approach relies heavily upon individuals to be able to intuit their true state of knowledge effectively, and such a feature holds in fields as seemingly diverse as nursing (Gobet and Chassy, 2008), and chess (Saariluoma, 2001, for instance, notes that expert players rapidly draw on 'thought models' that are relevant to the position of the game, activated by recognition, but able to be combined into more complex structures). There is, however, ample scope for intuition to be wrong, both in terms of experts being over-confident in their knowledge when there has been a change in system; and for novices lacking confidence but still having significant insights. Unfortunately, however, the dynamic issues that this raises are beyond the scope of this paper.

Scaling according to robustness of techniques for assessment

The second approach is to use discount factors that relate the state of knowledge to the perceived robustness of the evidence available. UK civil service estimates of the optimism bias that should be applied to business cases (New Economy, 2013: 26) provide the following indicative factors.

Table 4 Optimism bias correction factors for benefits by type of evidence

Evidence base for benefits	Age of data / analysis	Known data error	Optimism bias correction
Randomised Control Trial in UK	Current Data (<1 year old)	+-2%	0%
International Randomised Control Trial	1-2 years old	+-5%	-5%
Independent monitoring of outcomes with a robust evaluation plan	2-3 years old	+-10%	-10%
Practitioner monitoring of outcomes with a robust evaluation plan	3-4 years old	+-15%	-15%
Secondary evidence from a similar type of intervention	4-5 years old	+-20%	-25%
Uncorroborated expert judgement	>5 years old	+-25%	-40%

Source: New Economy (2013)

A comparison of tables 3 and 4 suggests a similar extent to which lack of knowledge leads to a discounting of value, and an implication of this is explored further in this paper.

Model incorporating state of knowledge

Both of the above approaches suggest that the non-financial returns will tend to be discounted by an investor, not only in line with the weighting between financial and non-financial returns per se; and not only to the extent that non-financial returns happen proportionately to the financial returns; but also with respect to the extent to which the non-financial returns are viewed as credible.

We adopt an equation in which expenditure A_{t-1} on assessment supplements existing knowledge (sk_{t-1}). This addition to knowledge process is scaled by the inclusion of a 'ratio of knowledge to cost' parameter θ, in the following equation (7): $sk_t = sk_{t-1}^{1-\theta A(t-1)/w(t-1)}$ where sk_t is between 0 and 1. This equation takes on board the effect of the ability to improve knowledge becoming harder in a non-linear way as the starting point for knowledge is higher.

A revised model incorporating knowledge into decisions has the form (8):

$$\text{Max } E_{t-1}\{p_{m\,t-1}\, q_{m\,t-1}\,(1+i) + p_{a\,t-1}\, q_{a\,t-1} * [(1 + d\, sk_t\, \alpha_{SER})\,(1 + \beta_r + \varepsilon_{rt})] - \delta / 2w_{t-1} * [\text{Var}\,(w_{t-1})]\}$$

The Lagrangean function is (9):

$$p_{m\,t-1}q_{m\,t-1}\,(1+i) + p_{a\,t-1}q_{a\,t-1} * [1 + d\,\alpha_{SER} * sk_{t-1}^{1-\theta A(t-1)/w(t-1)}]\,(1+\beta_r) - \delta / 2w_{t-1} * p_{a\,t-1}^2 q_{a\,t-1}^2 (1+ d\,\alpha_{SER})^2\,\sigma^2_R + \lambda * (w_{t-1} - p_{m\,t-1}\, q_{m\,t-1} - p_{a\,t-1}\, q_{a\,t-1} - A_{t-1})$$

It can be shown (see Annex 1) that the solution for $p_{a\,t-1}\, q_{a\,t-1} / w_{t-1}$ becomes a variant of (3):

$$10)\ p_{a\,t-1}\,\overline{q}_{a\,t-1} / w_{t-1} = \{(1+\beta_r)\,[1 + d\,\alpha_{SER} * sk_{t-1}^{1-\theta A(t-1)/w(t-1)}] - (1+i)\} / (\delta\sigma^2_R * (1 + d\,\alpha_{SER})^2).$$

Indicative parameters

The additional parameters introduced in this section are sk_{t-1} and θ. The value of sk_{t-1} is taken to be 0.333, reflecting basic initial thinking; and an estimate of θ of 22 is the best fit for a curve of this form to fit the values of sk_{t-1} and assessment costs as a proportion of wealth shown below in figure 1.

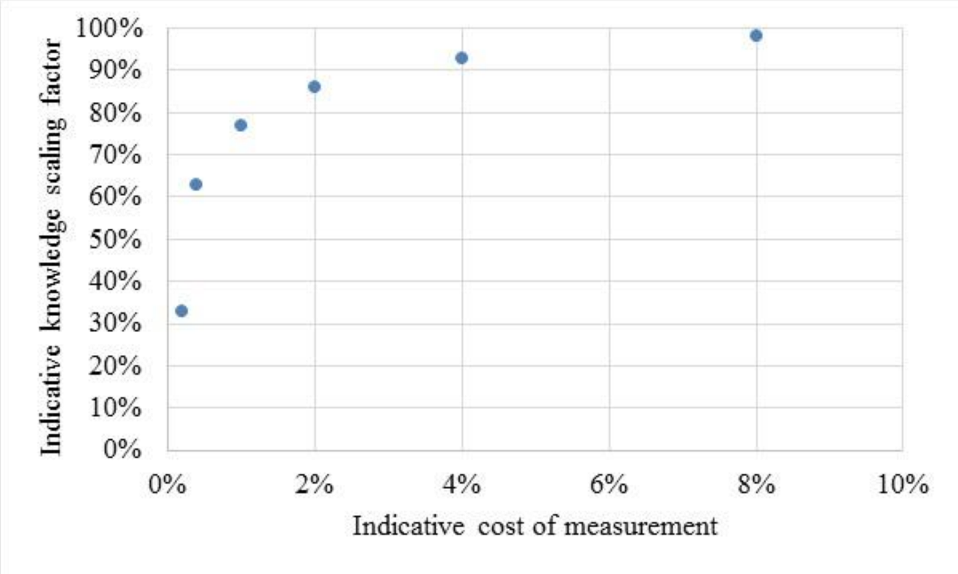

Figure 1 Indicative relationship between state of knowledge and assessment costs, using information theory coefficients

Table 5 shows shares of risky assets held under different rates of return under these parameters, and with an SER weighting of 5%. It also shows shares of assets held when the value of θ is halved to 11, reflecting a more difficult route to increasing knowledge.

Table 5 Assets held given different rates of return and different parameters for acquiring knowledge

	Zero level of risky assets	Full level of risky assets
[I] Expected financial rate of return (without assessment effects)	-1.96%	-1.21%
[II] Expected financial rate of return (standard level of θ)	-1.06%	-0.32%
[III] Expected financial rate of return (level of θ is halved)	-0.84%	-0.09%
[IV] Share of wealth in risky assets	0%	98%
[V] Share of wealth in risk free assets	98%	0%
[VI] Allocation on measurement	2%	2%

A comparison of rows I and II suggests that a higher rate of return is required (by an amount of the order of 0.9% per annum), when assessment effects are included – a significant level of change, given that the effect of risk aversion is to require a higher rate of return of the order of 0.7% per annum.

A comparison of the rows II and III also shows that more difficult measurement brings about a higher required financial rate of return (by an amount of the order of 0.2%). Hence, not only are some issues (such as the level of empowerment in a society) more difficult to measure, but also the level of SER demanded is higher when such intangibles are the subject of the programme.

Discussion

The model provides a highly simplified perspective on the issue of financial returns, non-financial returns and the state of knowledge of those non-financial returns. Such an initial foray into the field leaves many obvious potential refinements (increasing the number of assets with different characteristics; a more sophisticated version of the growth of knowledge; an equilibrium model of prices for assets; an allowance for different discount factors for the 'stock' of SER compared to the future 'flow' of SER to name four major agendas), but does suggest several clear hypotheses for comparison against empirical analysis:

- Those who put a weighting on SER are in theory prepared to accept a lower financial rate of return that can be as much as of the order of 1 or 2 percentage points p.a.;
- The lack of clear knowledge on SER performance reduces the extent to which investors would be willing to reduce financial returns by a high fraction of that potential margin, and by a greater effect than risk aversion per se;
- When the SER returns relate to intangibles that are harder to measure, there is a further significant reduction in the extent to which investors are willing to reduce financial returns.

These hypotheses appear to be in line with what practitioners have found. However, much more work is needed to develop a more sophisticated analysis and to develop effective databases with which to test the extent, if any, to which the hypotheses hold in practice.

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Annex Deriving key results

Base case model without assessment effects

Wealth is allocated between a risk-free asset and a risky asset relating to socially beneficial activities:

A1) $w_{t-1} - p_{m\,t-1}\, q_{m\,t-1} - p_{a\,t-1}\, q_{a\,t-1} = 0$

A2) $E_{t-1}[w_t] = p_{m\,t-1}\, q_{m\,t-1} * (1+ i) + p_{a\,t-1}\, q_{a\,t-1} * E_{t-1}[1 + r_t]$

The wider benefits for society are additive but discounted by d (knowing that social benefits of £1 have been achieved is generally worth only a fraction of that to an individual), and the extent to which the risky assets correlate with social benefits. This suggests an expectation function of the following form:

A3) $E[ww_t] = p_{m\,t-1}\, q_{m\,t-1} * (1+ i) + p_{a\,t-1}\, q_{a\,t-1} * E_{t-1}\{1 + d * \%\ SER_t \} * (1 + r_t)$

We adopt a standard utility function that increases in w_t, and declines with δ (the degree of risk aversion), multiplied by σ^2_w (the variance of wealth). For pure wealth effects the utility function is:

A4) $U (q_{m\,t-1}, q_{a\,t-1}) = E [w_t] – \delta * Var (w_{t-1}) / (2\, w_{t-1})$

The assumptions in respect of returns are that:

A5) $r_t = \beta_r + \varepsilon_{rt}$ where ε_R is distributed normally, mean 0, variance σ^2_R

A6) SER_t as a proportion of the value of risky assets is given by the term α_{SER}

A7) $cov (i_, r_t) = 0$, to exclude CAPM portfolio effects from the analysis.

The investor aims to maximise the expected utility of wider wealth, $E_{t-1}[U(ww_t)]$, through choice of $q_{m\,t-1}$ and $q_{a\,t-1}$. From A3 and A4, the problem can be expressed as:

A8) $Max\ E_{t-1}\, p_{m\,t-1}q_{m\,t-1}(1+i) + p_{a\,t-1}q_{a\,t-1} *(1+d\alpha_{SER}) * E_{t-1}[(1+\beta_r+\varepsilon_{rt})] - \delta/(2w_{t-1}) * E_{t-1}\, Var(w_{t-1})$

This can be simplified, for equilibrium choice ($\bar{q}_{m\,t-1}, \bar{q}_{a\,t-1}$) as follows:

A9) $Var(w_{t-1} | \bar{q}_{m\,t-1}, \bar{q}_{a\,t-1}) = Var ([1+ i] * p_{m\,t-1}\bar{q}_{m\,t-1}) + Var ([1+ r_t] * p_{a\,t-1}\, \bar{q}_{a\,t-1} * \{1+ d\alpha_{SER}\}) + 2\, Cov([1+ i] * p_{m\,t-1}\, \bar{q}_{m\,t-1}, [1+ r_t] * p_{a\,t-1}\bar{q}_{a\,t-1} * \{1 + d\, \alpha_{SER}\})$

$\simeq Var (1+ i)\, p_{m\,t-1}^2\, \bar{q}_{m\,t-1}^2 + Var (1+\beta_r+\varepsilon_{rt})\, p_{a\,t-1}^2\bar{q}_{a\,t-1}^2 (1+d\, \alpha_{SER})^2 +$
$2\, Cov ((1+i)*\bar{q}_{m\,t-1}\, p_{m\,t-1}, (1+\beta_r+\varepsilon_{rt})*(1+ d\, \alpha_{SER})\, p_{at-1}\bar{q}_{a\,t-1})$

since (1 + i) is fixed, and $Cov (i, \varepsilon_{rt}) = 0$ from A7, $Var (w_{t-1}) \simeq \sigma^2_R\ p_{a\,t-1}^2\, \bar{q}_{a\,t-1}^2 (1 + d\, \alpha_{SER})^2$.

The maximisation problem then equates to:

A10) $Max\ p_{m\,t-1}\, q_{m\,t-1}(1+ i) + p_{a\,t-1}\, q_{a\,t-1} *[1+ d\alpha_{SER}]*(1+\beta_r) - \delta/2w_{t-1}*p_{a\,t-1}^2 q_{a\,t-1}^2\sigma^2_R (1+d\, \alpha_{SER})^2$.

The Lagrangian takes this function and adds constraint (A1).

Differentiating with respect to choice of $q_{m\ t-1}$ and $q_{a\ t-1}$ and setting equal to zero, we obtain:

A11) $p_{m\ t-1}(1+i) + \lambda(-p_{m\ t-1}) = 0$, which implies that $\lambda = (1 + i)$,

A12) $p_{a\ t-1}(1 + d\ \alpha_{SER})(1 + \beta_r) - p_{a\ t-1}^2\ \overline{q}_{a\ t-1} * \delta\sigma^2_R (1 + d\ \alpha_{SER})^2 / w_{t-1} - (1+i)\ p_{a\ t-1} = 0$ (note that the second derivative in $q_{a\ t-1}$ is equal to $- p_{a\ t-1}^2\ \delta\sigma^2_R (1 + d\ \alpha_{SER})^2 / w_{t-1}$, which is negative for $\delta > 0$, hence this point represents a local maximum).

Rearranging for share of wealth in risky assets, $p_{a\ t-1}\ \overline{q}_{a\ t-1} / w_{t-1}$ we obtain:
A13) $p_{a\ t-1}\ \overline{q}_{a\ t-1} / w_{t-1} = \{1+ \beta_r\}/\{\delta\ \sigma^2_R [1 + d\ \alpha_{SER}]\} - (1+ i)/\{\delta\ \sigma^2_R [1 + d\ \alpha_{SER}]^2\}$

Model including assessment effects

We assume that the investor weights social benefits by a 'knowledge scaling factor' drawing on reports on social benefits, with the investor gaining deeper insights from additional efforts placed on assessment. We model the knowledge scaling factor by sk_t (which varies between 0 and 1), and model assessment expenditure by A_t. The link between sk_t ,sk_{t-1} and At-1 is given by A14): $E[sk_t] = sk_{t-1}^{\,1 - \theta\ A(t-1) / w(t-1)}$

The maximisation problem is therefore (A15):
Max $E_{t-1} \{p_{m\ t-1}\ q_{m\ t-1}(1+ i) + p_{a\ t-1}\ q_{a\ t-1} * [(1 + d\ sk_t\ \alpha_{SER})(1+\beta_r+\varepsilon_{rt})] - \delta / 2w_{t-1} * [Var(w_{t-1})]\}$

The Lagrangean is (A16): $p_{m\ t-1}\ q_{m\ t-1}(1+ i) + p_{a\ t-1}\ q_{a\ t-1} * [1 + d\ \alpha_{SER} * sk_{t-1}^{\,1 - \theta\ A(t-1) / w(t-1)}] (1+ \beta_r) - \delta / 2w_{t-1} * p_{a\ t-1}^2\ q_{a\ t-1}^{\ 2} (1 + d\ \alpha_{SER})^2\ \sigma^2_R + \lambda * (w_{t-1} - p_{m\ t-1}\ q_{m\ t-1} - p_{a\ t-1}\ q_{a\ t-1} - A_{t-1})$

The associated first order conditions, differentiating with respect to $q_{m\ t-1}$ and $q_{a\ t-1}$ are:

A17) $(1+ i)\ p_{m\ t-1} - \lambda\ p_{m\ t-1} = 0$, hence $\lambda = (1 + i)$

A18) $p_{a\ t-1}(1+\beta_r)[1 + d\alpha_{SER}*sk_{t-1}^{\,1-\theta\ A(t-1) / w(t-1)}] - p_{a\ t-1}^{\ 2}\overline{q}_{a\ t-1}(1+d\ \alpha_{SER})^2 * \delta\sigma^2_R / w_{t-1} - (1+i)p_{a\ t-1} = 0$

From (A18), and setting A_{t-1}/w_{t-1} to the value *a*, it follows that investment in risky assets is given by: A19) $p_{a\ t-1}\ \overline{q}_{a\ t-1} / w_{t-1} = \{(1 + \beta_r)\ [1 + d\ \alpha_{SER}\ sk_{t-1}^{\,1 - \theta a)}] - (1+ i)\}/(\delta\sigma^2_R * (1+d\ \alpha_{SER})^2)$

In assessing this equation, values of *a* are estimated from observed levels of expenditure on assessment by impact investors.