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QUANTITATIVE DESCRIPTION OF FINANCIAL TRANSACTIONS AND RISKS

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***Abstract**: This paper presents a quantitative model of financial transactions between economic agents on economic space. Risk ratings of economic agents play role of their coordinates. Aggregate amounts of agent's financial variables at point **x** define macro financial variables as functions of time and coordinates. Financial transactions between agents define evolution of agent's financial variables. Aggregate amounts of financial transactions between agents at points **x** and **y** define macro financial transactions as functions of **x** and **y**. Macro transactions determine evolution of macro financial variables. To describe dynamics and interactions of macro transactions we derive hydrodynamic-like equations. Description of macro transactions permits model evolution of macro financial variables and hence develop dynamics and forecasts of macro finance. As example for simple model interactions between macro transactions we derive hydrodynamic-like equations and obtain wave equations for their perturbations. Waves of macro transactions induce waves of macro financial variables on economic space. Diversities of financial waves of macro transactions and macro financial variables on economic space in simple models uncover internal complexity of macro financial processes. Any developments of financial models and forecast should take into account financial wave processes and their influences.*

***Keywords**: Macro Finance, Risk Ratings, Economic Space, Wave Equations*

JEL: C00, C02, C10, E00

Introduction

This paper describes financial transactions between economic agents those define evolution of financial variables. We regard agents as simple units of macro finance and use their risk ratings as their coordinates on economic space. Aggregate amounts of financial variables like Assets and Investment, Credits and Liabilities of agents at point ***x*** on economic space define corresponding macro finance variables as functions of time and coordinates ***x*** on economic space. Financial transactions like Investment or Credits between agents change their financial variables and thus define evolution of macro financial variables. Agents with particular risk rating ***x*** can Buy and Sell Assets, Invest and provide Credits to agents with any risk rating ***y***. That defines non-local, "action-at-a-distance" character of transactions between agents on economic space. Similar models of transactions between agents already exist in economics. Nearly eighty ears ago famous economist Leontief developed his input-output analysis or inter-industry tables framework (Leontief, 1936; 1941; 1973; Miller and Blair, 2009; Horowitz and Planting, 2009). In his Nobel Lecture Leontief (1973) indicates that: "Direct interdependence between two processes arises whenever the output of one becomes an input of the other: coal, the output of the coal mining industry, is an input of the electric power generating sector". Leontief allocates agents by Industry sectors and described transactions between Industries. We simply replace Leontief's allocations of agents by Industries and substitute it by allocation of agents by their risk ratings as coordinates on economic space. Leontief's framework aggregates input-output transactions of agents by Industries and establishes inter-industry tables. We aggregates financial transactions between agents at point ***x*** and point ***y*** on economic space and determine macro transactions on economic space as functions of two variables (***x,y***). Main advantage of our approach: allocation of agents by Industries does not define any space. Our approach introduces linear economic space that imbed allocation of agents by their risk ratings as coordinates. Usage of linear economic space enhances methods for economic and financial modeling.

Dynamics of macro transactions define evolution of macro variables on economic space. Parallels between agents as simple units of macro finance and multi-particles systems in physics permit derive hydrodynamic-like equations that describe macro transactions on economic space. For simple model relations between two macro transactions we derive hydrodynamic-like equations and then derive wave equations on macro transactions disturbances. Diversity of wave equations in simple models discovers complexity of internal relations between macro financial variables. Financial wave generation, propagation and interaction can play important role for macro finance modeling.

The rest of the paper is organized as follows. In Section 2 we argue model setup. In Section 3 we reconsider Leontief's input-output framework and define macro transactions that describe financial *"action-at-a-distance"* transactions between points ***x*** and ***y*** on economic space. In Section 4 we derive hydrodynamic-like equations that describe evolution and interactions between macro transactions. In Section 5 for simple interaction between Asset-Liabilities and Revenue-on-Assets macro transactions we derive hydrodynamic-like equations in a closed form. In Section 6 we derive wave equations on macro transaction's disturbances. Conclusions are in Section 7.

Model setup

Our model of macro finance uses well-known and familiar notions: economic agents, agent's risk ratings and Leontief's framework. Agents are primary units of any macro finance system. Each agent has many financial variables like Assets and Debts, Investment and Savings, Credits and Loans, and etc. Let call agents as "*independent*" if sum of extensive financial variables of any group of agents equals financial variable of entire group. For example: sum of Assets of *n* agent equals Assets of entire group. Let assume that all agents are "*independent*" and any extensive macro financial variable equal sum of corresponding financial variables of agents.

Economic space

Current financial models allocate agents by industries, financial sectors, by type of investors, and etc. We propose use ratings of agent's financial or economic risks as their coordinates (Olkhov, 2016a; 2016b; 2017a). International rating agencies (Fitch, 2006; S&P, 2011; Moody's, 2007) estimate risk ratings of huge corporations and banks and these ratings are widely used in finance. Due to current methodology risk ratings take values of finite number of risk grades like *AAA*, *BB*, *C* and etc. Let treat risk grades as points of discreet space and let call such a space as economic space. Let make following assumptions:

1. Let assume that rating agencies that estimate risk ratings for huge corporations and banks can also make assessment for small companies and even households – for all agents of macro financial system. Let treat finite number of risk grades as points of discreet economic space. Let treat risk ratings of agents as their coordinates on discreet economic space.

2. Let assume that generalization of risk assessment methodology may define continues risk grades that establish space *R*. Then risk ratings of agents can be treated as their coordinates on *R*.

3. Let assume that simultaneous risk ratings assessments of *n* economic or financial risks allow allocate agents on *n*-dimensional space that can be discreet or R^n.

Let define economic space as any mathematical space that is used to map agents by their risk ratings as space coordinates. Dimension of economic space is determined by number of different risks for which risk ratings are measured simultaneously. Let state that positive direction along each axis points to risk growth and negative direction points to risk decline. For brevity let call economic space as e-space and agents as economic particles or e-particles.

Definition of e-space uncovers many problems. Methodology of risk assessments should be extended to plot ratings of *n* different risks on R^n and to provide risk assessment for all economic agents of macro finance system. Definition of economic space with reasonable dimension *n* equals *two, three or four* requires selection of *two, three or four* risks responsible for major influence on macro financial processes. That permits establish economic space R^n with $n = 1,2,3$ dimensions and derive appropriate initial distributions of economic variables. To select most valuable risks one should establish procedures that compare influence of different risks on all agents. Selection of main risks simplifies macro finance model and allows neglect "small risks". Selections of major risks give opportunity to validate initial and target sets of risks and to prove or disprove initial model assumptions. It makes possible to compare predictions with observed financial data and outline causes of disagreements. It is well known that risks can suddenly arise and then vanish. To describe macro finance in a time term *T* one should forecast *m* main risks that will play major role in a particular time term and define economic space R^m. This set of *m* risks defines target state of e-space R^m. Transition from initial set of *n* main risk to target set of *m* risks describes evolution of initial representation R^n of to the target one R^m.

Let assume that we selected *n* major risks and determined risk ratings of all economic agents. Let assume that *n* major risks don't change and we can develop macro finance model on e-space R^n. Agent's risk ratings $\boldsymbol{x}$ play role of their coordinates $\boldsymbol{x}$ on economic space R^n. Thus it is possible define macro financial variables as functions of time *t* and coordinate $\boldsymbol{x}$. Let assume that agents are "*independent*" and hence sum of any extensive financial variables as Credits and Assets, Investment and Liabilities and etc., of agents at point $\boldsymbol{x}$ equal macro financial variable at point $\boldsymbol{x}$. For example, sum of Assets of all agents with coordinate $\boldsymbol{x}$ equals macro financial Assets at point $\boldsymbol{x}$. Agents at point $\boldsymbol{x}$ can perform financial transactions with agents at any point $\boldsymbol{y}$ on economic space. Financial transactions between agents "arise whenever the output of one becomes an input of the other" (Leontief, 1973). Let call financial transactions between all agents at points $\boldsymbol{x}$ and all agents at $\boldsymbol{y}$ as macro transaction or *financial field* that depends on coordinates $(\boldsymbol{x},\boldsymbol{y})$.

Macro financial variables

Let briefly explain reasons for transition from description of agent's variables to description of macro financial variables as functions of time *t* and coordinates $\boldsymbol{x}$ on economic space (Olkhov, 2016a, 2016b, 2017a – 2017d). Let complement widespread partition of agents by economic sectors and industries with partition of agents on economic space. Partitions of agents by economic sectors attribute Assets or Profits of Bank sector as cumulative Assets or Profits of all agents of this particular sector. Let replace common granularity by economic sectors and let allocate agents by their risk ratings $\boldsymbol{x}$ as coordinates $\boldsymbol{x}$ on economic space. Such allocation allows define macro financial variables as functions of $\boldsymbol{x}$ on economic space. Such transition has parallels to transition from description of multi-particle system in physics that takes into account granularity of separate particles to continuous media or hydrodynamic approximation. Indeed, risk ratings $\boldsymbol{x}$ of separate agents are changed under the action of financial processes and transactions between agents. Thus agents can move on economic space alike to "*economic gas*" and their motion can induce changes of agent's financial variables. For example random motion of agent on economic space can induce random changes of agent's Investment and Assets, Credits and Profits and etc. Let describe agents and their variables by probability distributions. Averaging of agent's financial variables by probability distributions allow describe macro finance alike to financial continuous media or financial hydrodynamic-like approximation. In such approximation we neglect granularity of variables like Assets or Capital that belong to separate agents at point $\boldsymbol{x}$ and describe Assets or Capital as function of $\boldsymbol{x}$ on economic space alike to "*Assets fluid*" or "*Capital fluid*" in hydrodynamics. In some sense such transition has parallels to partition of Assets by sectors or industries. The "small" difference: in common approach agents and their variables belong to permanent industry or sector. In our model agent's risk ratings define linear space and agents can move on economic space due to change of their risk ratings. These small distinctions cost a lot and allow model macro finance as a continuous "*financial media*".

Below for convenience we present definition of macro variables according to (Olkhov, 2016a, 2016b, 2017a). For brevity let further call agents as economic particles or e-particles and economic space as e-space. Let introduce macro variables at point $\boldsymbol{x}$ as sum of variables of e-particles with coordinates $\boldsymbol{x}$ on e-space.

Each e-particle has many financial variables like Assets and Debts, Investment and Savings, Credits and Loans, and etc. Let call e-particles as "*independent*" if sum of extensive (additive) variables of any group of e-particles equals variable of entire group. For example: sum of Assets of n e-particles equals Assets of entire group. Let assume that all e-particles are "*independent*" and any extensive macro financial variable equals sum of corresponding variables of agents. So, aggregation of Assets of e-particles with coordinates $\boldsymbol{x}$ on e-space define Assets as function of time t and $\boldsymbol{x}$. Integral of Assets by $d\boldsymbol{x}$ over e-space equals Assets of entire macro finance as function of time t. Coordinates of e-particles represent their risk ratings and hence they are under random motion on e-space. Thus sum of Assets of e-particles at point $\boldsymbol{x}$ also is random. To obtain regular values of macro variables like Assets at point $\boldsymbol{x}$ let average Assets at point $\boldsymbol{x}$ by probability distribution f. Let state that distribution f define probability to observe $N(\boldsymbol{x})$ e-particles with value of Assets equal $a_1,\dots a_{N(x)}$. That determine density of Assets at point $\boldsymbol{x}$ on e-space (Eq.(2.1) below). Macro Assets as function of time t and coordinate $\boldsymbol{x}$ behave alike to Assets fluid similar to fluids in hydrodynamics. To describe motion of Assets fluid (Olkhov, 2017a) let define velocity of such a fluid. Let mention that velocities of e-particles are not additive variables and their sum doesn't define velocity of Assets motion. To define velocities of Assets fluid correctly one should define "*Asset's impulses*" at point $\boldsymbol{x}$ as product of Assets a_j of particular j-e-particle and its velocity $\boldsymbol{v_j}$ (Eq. (2.2) below). Such "Asset's impulses" $a_j\,\boldsymbol{v_j}$ - are additive variables and sum of "Asset's impulses" can be averaged by similar probability distribution f. Densities of Assets and densities of Assets impulses permit define velocities of Assets fluid (Eq.(2.3) below). Different financial fluids can flow with different velocities. For example flow of Capital on e-space can have velocity higher then flow of Profits, nevertheless they are determined by motion of same e-particles. Let present these issues in a more formal way.

Let assume that each e-particle on e-space R^n at moment t is described by extensive variables $(u_1,\dots u_l)$. Extensive variables are additive and admit averaging by probability distributions. Intensive variables, like Prices or Interest Rates, cannot be averaged directly. Enormous number of extensive variables like Capital and Credits, Investment and Assets, Profits and Savings, etc., describe each e-particle and make financial modelling very complex. As usual, macro financial variables are defined as aggregate amounts of corresponding values of all e-particles of entire macro finance. For example, macro Investment equal aggregate

Investment and Assets can be calculated as cumulative Assets of all e-particles. Let define macro variables as functions of time t and coordinates $\boldsymbol{x}$ on e-space.

Let assume that there are $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$. Let state that velocities of e-particles at point $\boldsymbol{x}$ equal $\boldsymbol{v}=(\boldsymbol{v}_1,\dots \boldsymbol{v}_{N(x)})$. Each e-particle has l extensive variables $(u_1,\dots u_l)$. Let assume that values of variables equal $u=(u_{1i},\dots u_{li})$, $i=1,..N(\boldsymbol{x})$. Each extensive variable u_j at point $\boldsymbol{x}$ defines macro variable U_j as sum of variables u_{ji} of $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$

$$U_j = \sum_i u_{ji}\,; \quad j=1,..l\,; \quad i=1,\dots N(\boldsymbol{x})$$

To describe motion of variable U_j let establish additive variable alike to impulse in physics. For e-particle i let define impulses p_{ji} as product of extensive variable u_j that takes value u_{ji} and its velocity $\boldsymbol{v}_i$:

$$p_{ji} = u_{ji}\boldsymbol{v}_i \tag{1.1}$$

For example if Assets a of e-particle i take value a_i and velocity of e-particle i equals $\boldsymbol{v}_i$ then impulse pa_i of Assets of e-particle i equals $pa_i = a_i{\cdot}\boldsymbol{v}_i$. Thus if e-particle has l extensive variables $(u_1,\dots u_l)$ and velocity $\boldsymbol{v}$ then it has l impulses $(p_1,p_2,..p_l)=(u_1{\cdot}\boldsymbol{v},\dots u_l{\cdot}\boldsymbol{v})$. Let define impulse P_j of variable U_j as

$$\boldsymbol{P}_j = \sum_i u_{ji}\cdot\boldsymbol{v}_i\,; \quad j=1,..l\,; \quad i=1,\dots N(\boldsymbol{x}) \tag{1.2}$$

Let introduce distribution function $f=f(t,x;U_1,..U_l,\boldsymbol{P}_1,..\boldsymbol{P}_l)$ that determine probability to observe variables U_j and impulses $\boldsymbol{P}_j$ at point $\boldsymbol{x}$ at time t. U_j and $\boldsymbol{P}_j$ are determined by corresponding values of e-particles that have coordinates $\boldsymbol{x}$ at time t. They take random values at point $\boldsymbol{x}$ due to random motion of e-particles on e-space. Averaging of U_j and $\boldsymbol{P}_j$ within distribution function f allows establish transition from approximation that takes into account variables of separate e-particles to continuous "*financial media*" or hydrodynamic-like approximation that neglect e-particles granularity and describe averaged macro financial variables as functions of time and coordinates on e-space. Let define density functions

$$U_j(t,\boldsymbol{x}) = \int U_j\, f(t,\boldsymbol{x},U_1,\dots U_l,\boldsymbol{P}_1,..\boldsymbol{P}_l)\, dU_1..dU_l d\boldsymbol{P}_1..d\boldsymbol{P}_l \tag{2.1}$$

and impulse density functions $\boldsymbol{P}_j(t,x)$

$$\boldsymbol{P}_j(t,\boldsymbol{x}) = \int \boldsymbol{P}_j\, f(t,\boldsymbol{x},U_1,\dots U_l,P_1,..P_l)\, dU_1..dU_l d\boldsymbol{P}_1..d\boldsymbol{P}_l \tag{2.2}$$

That allows define e-space velocities $\boldsymbol{v}_j(t,\boldsymbol{x})$ of densities $U_j(t,\boldsymbol{x})$ as

$$U_j(t,\boldsymbol{x})\boldsymbol{v}_j(t,\boldsymbol{x}) = \boldsymbol{P}_j(t,\boldsymbol{x}) \tag{2.3}$$

Densities $U_j(t,\boldsymbol{x})$ and impulses $\boldsymbol{P}_j(t,\boldsymbol{x})$ are determined as mean values of aggregate amounts of corresponding variables of separate e-particles with coordinates $\boldsymbol{x}$. Functions $U_j(t,\boldsymbol{x})$ can describe macro densities of Investment and Loans, Assets and Debts and so on.

To describe evolution of macro variables like Investment and Loans, Assets and Debts and etc., let remind that they are composed (Eq. 2.1-2.3) by corresponding variables of e-particles. However Assets of e-particle 1 at point $\boldsymbol{x}$ are determined by numerous Buy or Sell transactions of Assets from e-particles at any points $\boldsymbol{y}$ on e-space. To describe evolution of macro variables let introduce and describe macro transactions on e-space.

Macro transactions

To change its Assets e-particle should Buy or Sell them. Value of Assets of e-particle can change due to variations of market prices determined by Buy-Sell market transactions performed by other e-particles. Any e-particles at point $\boldsymbol{x}$ can carry out transactions with e-particles at any point $\boldsymbol{y}$ on e-space.

Macro variables like Assets, Investment or Credits and etc., have important property. For example macro Investment at moment *t* determine Investment made during certain time term *T* that may be equal minute, day, quarter, year and etc. Thus any variable at time *t* is determined by factor *T* that indicates time term of accumulation of that variable. The same parameter *T* defines duration of transaction. Let further treat any transactions as rate or speed of change of corresponding variable. For example let treat transactions by Investment at moment *t* as Investment made during time term *dt*.

Financial transactions between e-particles are the only tools that implement financial interactions and processes. In his Nobel Lecture Leontief (1973) indicates that: "Direct interdependence between two processes arises whenever the output of one becomes an input of the other: coal, the output of the coal mining industry, is an input of the electric power generating sector". Let call financial variables of two e-particles as *mutual* if "the output of one becomes an input of the other". For example, Credits as output of Banks are *mutual* to Loans as input of Borrowers. Assets as output of Investors are *mutual* to Liabilities as input of Debtors. Any exchange between e-particles by *mutual* variables is carried out by corresponding transaction. Transactions between two e-particles at points $\boldsymbol{x}$ and $\boldsymbol{y}$ by Assets, Liabilities, Capital, Investment and etc., define function of time *t* and variables $(\boldsymbol{x},\boldsymbol{y})$. Different transactions define evolution of different couples of m*utual* variables. Let repeat that above treatment has parallels to Leontief's framework. We replace Leontief's allocation of agents by industries with mapping agents on e-space. Thus we replace transactions between industries - inter-industry tables - with transactions between points on e-space: by macro financial transactions between points $(\boldsymbol{x},\boldsymbol{y})$ on e-space. And most important distinction: inter-industry tables do not allow develop time evolution of macro finance because in reality coefficients matrix between different industries are not constant and are not described by Leontief's framework. As we show below, our approach gives ground for macro financial modelling by hydrodynamic-like equations on macro transactions.

Let call that financial transactions between e-particle *1* at point $\boldsymbol{x}$ and e-particle *2* at point $\boldsymbol{y}$ determine financial field $a_{1,2}(\boldsymbol{x},\boldsymbol{y})$ that describes exchange of variables $B_{out}(1,\boldsymbol{x})$ and $B_{in}(2,\boldsymbol{y})$ and at moment *t* during time term *dt*. Let $a_{1,2}(\boldsymbol{x},\boldsymbol{y})$ be equal to output variable $B_{out}(1,\boldsymbol{x})$ from e-particle *1* to e-particle *2* and equal to input of variable $B_{in}(2,\boldsymbol{y})$ of e-particle *2* from e-particle *1* at moment *t* during time term *dt*. So, $a_{1,2}(\boldsymbol{x},\boldsymbol{y})$ describes speed of change of variable $B_{out}(1,\boldsymbol{x})$ of e-particle *1* at point $\boldsymbol{x}$ due to exchange with e-particle *2* at point $\boldsymbol{y}$. The same time $a_{1,2}(\boldsymbol{x},\boldsymbol{y})$ describes speed of change of variable $B_{in}(2,\boldsymbol{y})$ of e-particle *2* at point $\boldsymbol{y}$ due to exchange with e-particle *1*. Thus variable $B_{out}(1,\boldsymbol{x})$ of e-particle *1* at point $\boldsymbol{x}$ changes due to action of financial field $a_{1,2}(\boldsymbol{x},\boldsymbol{y})$ with all e-particles at point $\boldsymbol{y}$ as follows:

$$dB_{out}(1,\boldsymbol{x}) = \sum_i a_{1,i}(\boldsymbol{x},\boldsymbol{y})\, dt \tag{3.1}$$

and vice versa

$$dB_{in}(2,\boldsymbol{y}) = \sum_i a_{i,2}(\boldsymbol{x},\boldsymbol{y})\, dt \tag{3.2}$$

For example Credits-Loans financial field may describe Credits (output) from e-particle *1* to e-particle *2*. For such a case $B_{in}(2)$ equals Loans received by e-particle *2* and $B_{out}(1)$ equals Credits issued by e-particle *1* during certain time term *T*. Sum of financial field over all input e-particles equals speed of change of output variable $B_{out}(1)$ of e-particle *1*.

Let assume that all extensive variables of e-particles can be presented as pairs of *mutual* variables or can be describes by *mutual* variables. Otherwise there should be macro variables that don't depend on any economic or financial transactions, don't depend on Markets, Investment and etc. We assume that any financial variable of e-particles depends of certain transactions between e-particles. For example Value of e-particle (Value of Corporation or Bank) don't take part in transactions but is determined by market transactions that define of stock price of corresponding Bank or by variables like Assets and Liabilities, Credits and Loans, Sales and Purchases and etc. Let assume that all extensive variables can be described by Eq.(3.1,3.2) or through other *mutual* variables. Thus macro transactions describe all extensive variables of e-particles and hence determine evolution of macro finance.

Now let explain transition from description of transactions between e-particle to description of macro transactions between points on e-space. Let assume that transactions between e-particles at point $\boldsymbol{x}$ and e-particles at point $\boldsymbol{y}$ are determined by exchange of *mutual* variables like Assets and Liabilities, Credits and Loans, Buy and Sell, and etc. Different transactions describe exchange by different *mutual* variables. For example *Assets-Liabilities (al)* transactions at time *t* describe a case when e-particle "one" at point $\boldsymbol{x}$ during time *dt* Invest (output) into Assets of amount *al* of e-particle "two" at point $\boldsymbol{y}$ and e-particle "two" at point $\boldsymbol{y}$ at time *t* during time *dt* receives Investment (input) that increase its *Liabilities* on amount *al* in front of e-particle "one" at point $\boldsymbol{x}$. Let give formal definition of macro transactions based on example of *Assets-Liabilities* transactions.

As above let assume that macro finance is under action of *n* major risks and each e-particle on e-space R^n at moment *t* is described by coordinates $\boldsymbol{x}=(x_1,\dots x_n)$ and velocities $\boldsymbol{\upsilon}=(\upsilon_1,\dots\upsilon_n)$. Let assume that at moment *t* there are $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$ and $N(\boldsymbol{y})$ e-particles at point $\boldsymbol{y}$. Let state that velocities of e-particles at point $\boldsymbol{x}$ equal $\boldsymbol{\upsilon=(\upsilon_1,\dots\upsilon_{N(x)})}$. Let state that at moment *t* each of $N(\boldsymbol{x})$ e-particles at point $\boldsymbol{x}$ carry Assets-Liabilities transactions $al_{i,j}(\boldsymbol{x,y})$ with e-particles $N(\boldsymbol{y})$ at point $\boldsymbol{y}$. In other words, if e-particle *i* at moment *t* at point $\boldsymbol{x}$ allocates its Assets by $al_{i,j}(\boldsymbol{x,y})$ at e-particle *j* at point $\boldsymbol{y}$ then e-particle particle *j* at point $\boldsymbol{y}$ at moment *t* increases its Liabilities by $al_{i,j}(\boldsymbol{x,y})$ in front of e-particle *i*. Let assume that all e-particles on e-space are "independent" and thus sum by *i* of Assets-Liabilities transactions $al_{i,j}(\boldsymbol{x,y})$ at point $\boldsymbol{x}$ on e-space R^n at time *t* during *dt* equal rise of Liabilities $l_j(\boldsymbol{x,y})$ of e-particle *j* at point $\boldsymbol{y}$ in front of all e-particles at point $\boldsymbol{x}$ at moment *t*

$$l_j(\boldsymbol{x},\boldsymbol{y}) = \sum_i al_{ij}(\boldsymbol{x},\boldsymbol{y}) = a_j(\boldsymbol{x},\boldsymbol{y}); \quad i = 1,\dots N(\boldsymbol{x}); \quad j = 1,\dots N(\boldsymbol{y})$$

and equal rise $a_j(\boldsymbol{x,y})$ of Assets at moment *t* during *dt* of all e-particles at point $\boldsymbol{x}$ allocated at e-particle *j* at point $\boldsymbol{y}$. Sum by *j* of transactions $al_{i,j}(\boldsymbol{x,y})$ at point $\boldsymbol{y}$ on e-space R^n equals rise $a_i(\boldsymbol{x,y})$ of Assets of e-particle *i* at point $\boldsymbol{x}$ allocated at all e-particles at point $\boldsymbol{y}$

$$a_i(\boldsymbol{x},\boldsymbol{y}) = \sum_j al_{ij}(\boldsymbol{x},\boldsymbol{y}) = l_i(\boldsymbol{x},\boldsymbol{y}); \; i = 1,\dots N(\boldsymbol{x}); j = 1,\dots N(\boldsymbol{y})$$

and equals rise of Liabilities of all e-particles at point $\boldsymbol{y}$ in front of e-particle *i* at point x. Let define transactions $al(\boldsymbol{x,y})$ between points $\boldsymbol{x}$ and $\boldsymbol{y}$ as

$$al(\boldsymbol{x},\boldsymbol{y}) = \sum_{ij} al_{ij}(\boldsymbol{x},\boldsymbol{y}); \quad i = 1,\dots N(\boldsymbol{x}); \; i = 1,\dots N(\boldsymbol{y}) \tag{4.1}$$

*al(**x**,**y**)* equals growth of Assets of all e-particles at point ***x*** that are allocated at e-particles at point ***y*** at moment *t* and equals rise of Liabilities of all e-particles at point ***y*** in front of all e-particles at point ***x*** at moment *t*. Transactions (4.1) between two points on e-space are random due to random character of deals between e-particles. To introduce transactions as regular function and to derive equations that describe evolution of regular macro transactions on e-space let introduce equivalent of "*transaction's impulse*" alike to Eq.(1.1, 1.2) and (Olkhov, 2017a, 2017c; 2017d). To do that let define additive variables $\boldsymbol{p_X}$ and $\boldsymbol{p_Y}$ that describe flux of Assets by e-particles along ***x*** and ***y*** axes. For Assets-Liabilities transactions *al* let define *impulses* $\boldsymbol{p}$ =($\boldsymbol{p_X}$, $\boldsymbol{p_Y}$) alike to Eq.(1.1; 1.2)

$$\boldsymbol{p_X} = \sum_{i,j} al_{ij} \cdot \boldsymbol{v_i}\,; \quad i = 1, \dots N(\boldsymbol{x}); j = 1, \dots N(\boldsymbol{y}) \tag{4.2}$$

$$\boldsymbol{p_Y} = \sum_{i,j} al_{ij} \cdot \boldsymbol{v_j}\,; \quad i = 1, \dots N(\boldsymbol{x}); j = 1, \dots N(\boldsymbol{y}) \tag{4.3}$$

Assets-Liabilities transactions *al(t,**x**,**y**)* (4.1) and "impulses" $\boldsymbol{p_X}$ and $\boldsymbol{p_Y}$ (4.2, 4.3) take random values due to random motion of e-particles. To obtain regular functions let apply averaging procedure. Let introduce distribution function $f=f(t, \boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}); al, \boldsymbol{p}=(\boldsymbol{p_X},\boldsymbol{p_Y}))$ on 2*n*-dimensional e-space R^{2n} that determine probability to observe Assets-Liabilities financial field *al* at point $\boldsymbol{z}=(\boldsymbol{x}, \boldsymbol{y})$ with impulses $\boldsymbol{p}$ =($\boldsymbol{p}_X$, $\boldsymbol{p}_Y$) at time *t*. Averaging of Assets-Liabilities transactions and their "impulses" within distribution function *f* determine "mean" continuous financial media or financial hydrodynamic-like approximation of transactions as functions of $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$. Let call hydrodynamic-like approximations of transactions as macro transactions. Assets-Liabilities financial field $AL(\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ and "impulses" $\boldsymbol{P}=(\boldsymbol{P_X},\boldsymbol{P_Y})$ take form:

$$AL(t, \boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y})) = \int al\, f(t, \boldsymbol{x}, \boldsymbol{y}; al, \boldsymbol{p_X}, \boldsymbol{p_Y})\, dal\, d\boldsymbol{p_X}\, d\boldsymbol{p_Y} \tag{5.1}$$

$$\boldsymbol{P_X}(t, \boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y})) = \int \boldsymbol{p_X}\, f(t, \boldsymbol{x}, \boldsymbol{y}; al, \boldsymbol{p_X}, \boldsymbol{p_X})\, dal d\boldsymbol{p_X} d\boldsymbol{p_Y} \tag{5.2}$$

$$\boldsymbol{P_Y}(t, \boldsymbol{z} = (\boldsymbol{x}, \boldsymbol{y})) = \int \boldsymbol{p_Y}\, f(t, \boldsymbol{x}, \boldsymbol{y}; al, \boldsymbol{p_X}, \boldsymbol{p_Y})\, dal\, d\boldsymbol{p_X} d\boldsymbol{p_Y} \tag{5.3}$$

That defines e-space velocity $\boldsymbol{v}(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))=(\boldsymbol{v}_x(t,\boldsymbol{z}),\boldsymbol{v}_y(t,\boldsymbol{z}))$ of financial field $AL(t, \boldsymbol{z})$:

$$\boldsymbol{P_X}(t, \boldsymbol{z}) = AL(t, \boldsymbol{z})\boldsymbol{v_X}(t, \boldsymbol{z}) \tag{5.4}$$

$$\boldsymbol{P_Y}(t, \boldsymbol{z}) = AL(t, \boldsymbol{z})\boldsymbol{v_Y}(t, \boldsymbol{z}) \tag{5.5}$$

Macro transactions may describe many important properties. Assets-Liabilities field $AL(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ describes distribution of rate of Investment made from point ***x*** (from agents with risk rating ***x***) to point ***y*** (to agents with risk ratings ***y***) at moment *t* during time term *dt.* Due to Eq.(2.1) integral of financial field *AL(**x**,**y**)* by variable ***y*** over e-space R^n defines rate of Investment from point ***x***. Integral of *AL(**x**,**y**)* by ***x*** over e-space R^n determines speed of change of total Investment made at point ***y*** or Liabilities at point ***y*** in front of all e-particles of entire economics. Integral of *AL(t,**x**,**y**)* by variables ***x*** and ***y*** on e-space describes function *A(t)* that equals rate of growth or decline of total Assets in economics or rate of change of total Liabilities. We simplify the problem and treat transactions between e-particles as only tool for implementation of financial processes. Meanwhile Credits-Loans field *CL(**x**,**y**)* define Credits landing from point ***x*** to point ***y*** at moment *t* during time term *dt*. Integral of *CL(**x**,**y**)* by variable ***y*** over e-space defines speed of Credits allocation from all e-particles at point ***x***. Integral of *CL(**x**,**y**)* by ***y*** over e-space determines speed of Loans change at point ***y***. Integral of *CL(**x**,**y**)* by ***x*** and ***y*** over e-space defines total Credits *C(t)* provided at moment *t* or total Loans received. Credits-Loans field *CL(**x**,**y**)* can determine position of maximum Creditors at point $\boldsymbol{x_C}$ and position $\boldsymbol{y_B}$ of maximum Borrowers of Credits and distance between them. Assets-Liabilities field *AL(**x**,**y**)* can define position of maximum Assets at point $\boldsymbol{x_A}$ and position of maximum

Liabilities at point $\boldsymbol{y}_L$ and describe dynamics of distance between these points. These relations could be very important for financial modelling. Below we derive hydrodynamic-like equations to describe evolution of Assets-Liabilities macro transactions.

Hydrodynamic-like equations

Macro transactions that define transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ on e-space describe evolution of macro financial variables. To describe macro transactions let derive hydrodynamic-like equations alike to (Olkhov, 2016a, 2017a, 2017c). Financial meaning and reasons for usage of hydrodynamic-like equations are very clear and simple. To describe evolution of financial field $A(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ and its impulses $\boldsymbol{P}=(\boldsymbol{P}_x,\boldsymbol{P}_y)=(\boldsymbol{v}_xA,\boldsymbol{v}_yA)$ in unit volume dV at point $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$ on $2n$-dimension e-space R^{2n} one should take into account two factors. First factor describes evolution of financial field $A(t,\boldsymbol{z})$ in unit volume due to change in time as $\partial A/\partial t$ and due to flux $\boldsymbol{v}\cdot A$ of financial field through surface of unit volume. Such flux is described by divergence from unit volume and equals div($\boldsymbol{v}\cdot A(t,\boldsymbol{z})$). Here $\boldsymbol{v}$ – velocity of financial field $A(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ on $2n$-dimension e-space R^{2n}. So, first factor defines left side of hydrodynamic-like equations. Second factor describes impact of other macro transactions or any other causes on financial field A and define right side of hydrodynamic-like equations. The same meaning have hydrodynamic-like equations on impulses $\boldsymbol{P}=(\boldsymbol{P}_x,\boldsymbol{P}_y)=(\boldsymbol{v}_xA,\boldsymbol{v}_yA)$ of financial field. For simplicity equations on impulses take form of Equation of Motion on velocity $\boldsymbol{v}=(\boldsymbol{v}_x,\boldsymbol{v}_y)$ of financial field A. Below we present these considerations in a more formal way.

Financial field $A(t,\boldsymbol{x},\boldsymbol{y})$ and impulses $\boldsymbol{P}(t,\boldsymbol{x},\boldsymbol{y})$ are determined in (5.1-5.5) by averaging procedures of aggregates of Assets-Liabilities transactions between e-particles at points $\boldsymbol{x}$ and $\boldsymbol{y}$. Similar macro transactions can describe *mutual* variables as Credits and Loans transactions, Buy and Sell transactions and etc. Let define field $A(\boldsymbol{x},\boldsymbol{y})$ between two *mutual* variables $A_{out}(\boldsymbol{x})$ and $A_{in}(\boldsymbol{y})$ on e-space R^n. $A(\boldsymbol{x},\boldsymbol{y})$ equals input $A_{in}(\boldsymbol{y})$ at $\boldsymbol{y}$ from $\boldsymbol{x}$ and equals output $A_{out}(\boldsymbol{x})$ from $\boldsymbol{x}$ to $\boldsymbol{y}$. Functions $A(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))$ and $\boldsymbol{v}(t,\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y}))=(\boldsymbol{v}_x(t,\boldsymbol{z})),\boldsymbol{v}_y(t,\boldsymbol{z}))$ are determined on $2n$-dimensional e-space R^{2n}. Similar to (Olkhov, 2016a, 2017a; 2017c) Continuous Equations (6.1) and Equations of Motion (6.2) on $A(t,\boldsymbol{z})$ take form:

$$\frac{\partial A}{\partial t} + div(\boldsymbol{v}A) = Q_1 \tag{6.1}$$

$$A\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = \boldsymbol{Q}_2 \tag{6.2}$$

Let repeat economic meaning of equations (6.1, 6.2). Left side of Eq.(6.1) describes change of $A(t,\boldsymbol{z})$ in unit volume on e-space R^{2n} at point $\boldsymbol{z}=(\boldsymbol{x},\boldsymbol{y})$. It can change due to variations in time that are described by derivative $\partial A/\partial t$ and due to flux $A(t,\boldsymbol{z})\boldsymbol{v}$ through surface of unit volume that is equal to $div(A\boldsymbol{v})$. Q_1 describe external factors like other macro transactions that can change $A(t,\boldsymbol{z})$. Left side of Equations of Motion describes same variation of field's impulse $\boldsymbol{P}(t,\boldsymbol{z}) = A(t,\boldsymbol{z})\boldsymbol{v}(t,\boldsymbol{z})$. Taking into account Continuity Equations left side of Equations of Motion can be simplified and take form (6.2). $\boldsymbol{Q}_2$ describe any factors that can change left side *(6.2)*.

Eq.(6.1; 6.2) on field $A(t,\boldsymbol{z})$ and its velocity $\boldsymbol{v}(t,\boldsymbol{z})$ are determined by factors Q_1 and $\boldsymbol{Q}_2$. Let assume that macro transactions $B(t,\boldsymbol{z})$ different from $A(t,\boldsymbol{z})$ define Q_1 and $\boldsymbol{Q}_2$. Let call these macro transactions $B(t,\boldsymbol{z})$ are *conjugate* to field $A(\boldsymbol{z})$ if $B(t,\boldsymbol{z})$ or their velocities determine right hand side factors Q_1 and $\boldsymbol{Q}_2$ of hydrodynamic-like equations (6.1; 6.2). Any field $A(t,\boldsymbol{z})$ can have one, two or many *conjugate* macro transactions $B(t,\boldsymbol{z})$ that determine right hand side of (6.1; 6.2). For example, Assets-Liabilities field may depend on Revenue-on-Assets field, Buy-Sell macro transactions can be determined by transactions with various Assets and etc. Credits-Loans field may depend on Payment-on-Credits field, Supply-Demand or Buy-Sell macro transactions defined by transactions with commodities and etc. In the simplest approximation let assume that field *A(t,**z**)* has only one conjugate field *B(t,**z**)* and vice versa. For that case it is possible to derive hydrodynamic-like equations on macro transactions *A(t,**z**)* and *B(t,**z**)* in a closed form and study their evolution under their mutual interactions. As example, let define Revenue-on-Assets field and study simplest model of mutual dependence between Assets-Liabilities and Revenue-on-Assets macro transactions.

Two conjugate macro transactions model

To derive Eq.(6.1; 6.2) in a closed form let study simplest model of mutual dependence between two *conjugate* macro transactions as Assets-Liabilities *AL(**z**)* and Revenue-on-Assets *RA(**z**)*. Let define Revenue-on-Assets *RA(**z**=(**x**,**y**))* field as all payoffs that are made by e-particles at point ***y*** in front of their Liabilities against Investors at point ***x*** that have allocated their Assets at ***y***. Thus Revenue-on-Assets field *RA(**z**=(**x**,**y**))* describes Income from point ***y*** to point ***x*** at moment *t* during time term *dt*. Field *AL(**z**=(**x**,**y**))* describes Assets allocations from point ***x*** to point ***y*** at moment *t*. Assets-Liabilities *AL(**z**)* and Revenue-on-Assets *RA(**z**)* macro transactions describe core financial properties. These transactions are responsible for growth and financial sustainability and their descriptions are extremely complex. Introduction of e-space allows establish and study various models that describe relations between variables and macro transactions and model different approximations of real financial processes.

Let start with simple model and assume that Assets-Liabilities field *Al(t,**z**=(**x**,**y**))* at moment *t* depends on Revenue-on-Assets field *RA(t,**z**=(**x**,**y**))* at moment *t* only. Our assumptions mean that Investors at point ***x*** take decisions on Assets allocations to point ***y*** on base of Revenue-on-Assets received from point ***y*** to point ***x*** at same moment *t*. We simplify the problem to develop reasonable model of their mutual interaction. To describe evolution of Assets-Liabilities field *AL(t,**z**)* let take Eq.(6.1; 6.2) and define factors Q_1 and $\boldsymbol{Q}_2$ using same approach and considerations as (Olkhov, 2016a, 2017a). Let assume that Q_1 on the right hand side of Continuity Equation (6.1) for Assets-Liabilities field *AL(t,**z**)* is proportional to divergence of Revenue-on-Assets velocity ***u(z)*** on e-space R^{2n}:

$$Q_1 \sim RA(\boldsymbol{z})\nabla\cdot\boldsymbol{u}(\boldsymbol{z}) \tag{7.1}$$

Positive divergence (7.1) of Revenue-on-Assets *RA(**z**)* field velocity ***u**(t,**z**)* describes growth of flux of Revenue-on-Assets and that may attract Investors at point ***x*** to increase their Assets at point ***y***. Negative divergence of velocity ***u**(t,**z**)* means that Revenue-on-Assets *IA(**z**)* flow decrease and that may prevent Investors at point ***x*** from further Assets allocations at point ***y***. Let assume that Q_1 factor that defines right hand side of (6.1) for Revenue-on-Assets field *RA(t,**z**)* is proportional to divergence of Assets-Liabilities velocity ***v**(t,**z**)*:

$$Q_1 \sim AL(\boldsymbol{z})\nabla\cdot\boldsymbol{v}(\boldsymbol{z}) \tag{7.2}$$

Positive divergence (7.2) of Assets-Liabilities *AL(**z**)* field velocity ***v**(t,**z**)* describes growth of Assets-Liabilities flux and that may increase Revenue-on-Assets *RA(t,**z**)*: growth of Investment from point ***x*** to point ***y*** on e-space may induce growth of payoffs on Assets from ***y*** to ***x***. As well negative divergence of Assets-Liabilities *AL(**x**,**y**)* flux describes decline of Assets flow allocated by point ***x*** at ***y*** and that may reduce payoffs on Assets from ***y*** to ***x***. It is obvious that we neglect time gap between Assets allocations and Revenue-on-Assets and other factors that may determine Investment decisions from ***x*** to ***y*** to simplify the model. Let determine $\boldsymbol{Q}_2$ factors in Equations of Motion (6.2) for Assets-Liabilities field *AL(**z**=(**x**,**y**))*. Let assume that velocity ***v**(t,**z**)* of Assets-Liabilities field *AL* depends on right hand side factor $\boldsymbol{Q}_2$ that is proportional to gradient of Revenue-on-Assets *RA(t,**z**)*:

$$\boldsymbol{Q}_2 \sim \nabla\, RA(\boldsymbol{z}) \tag{7.3}$$

Relations (7.3) propose that Assets-Liabilities field velocity $\boldsymbol{v}(t,\boldsymbol{z})$ grows in direction of higher Revenue-on-Assets. Let make same assumptions on $\boldsymbol{Q}_2$ that determines Equation of Motion (6.2) for Revenue-on-Assets field velocity $\boldsymbol{u}(t,\boldsymbol{z})$:

$$\boldsymbol{Q}_2 \sim \nabla AL(\boldsymbol{z}) \tag{7.4}$$

Relations (7.4) propose that Revenue-on-Assets field velocity $\boldsymbol{u}(t,\boldsymbol{z})$ grows up in the direction of higher Assets-Liabilities. Assumptions (7.1-7.4) define right hand side factors and define hydrodynamic-like equations for two *conjugate* macro transactions Assets-Liabilities and Revenue-on-Assets in a closed form. Continuity Equations:

$$\frac{\partial AL}{\partial t} + \nabla\cdot(\boldsymbol{v}AL) = a_2 RA(\boldsymbol{z})\nabla\cdot\boldsymbol{u}(\boldsymbol{z}) \tag{8.1}$$

$$\frac{\partial RA}{\partial t} + \nabla\cdot(\boldsymbol{u}RA) = a_1 AL(\boldsymbol{z})\nabla\cdot\boldsymbol{v}(\boldsymbol{z}) \tag{8.2}$$

Equations of Motion:

$$AL(\boldsymbol{z})\left[\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v}\cdot\nabla\boldsymbol{v}\right] = b_2\nabla RA(\boldsymbol{z}) \tag{8.3}$$

$$RA(\boldsymbol{z})\left[\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right] = b_1\nabla AL(\boldsymbol{z}) \tag{8.4}$$

Equations (8.1-8.4) give ground for derivation of financial wave equations.

Financial wave equations

Let derive equations on field's disturbances in linear approximation. Let simplify the problem and assume

$$AL(\boldsymbol{z}) = AL + al(\boldsymbol{z})\;;\; RA(\boldsymbol{z}) = RA + ra(\boldsymbol{z}) \tag{9.1}$$

Let assume that *AL* and *RA* are constant or their variations are negligible to compare with variations of small disturbances *al(**z**), ra(**z**), **v**(**z**) and **u**(**z**)* and let neglect nonlinear factors in Eq.(8.1-8.4). These assumptions allow derive equation on disturbances in linear approximation *alike to* derivation of acoustic wave equations (Landau and Lifshitz, 1987). Continuity Equations on disturbances take form:

$$\frac{\partial al}{\partial t} + AL\nabla\cdot\boldsymbol{v} = \alpha_2 RA\nabla\cdot\boldsymbol{u} \quad ; \quad \frac{\partial ra}{\partial t} + RA\nabla\cdot\boldsymbol{u} = \alpha_1 AL\nabla\cdot\boldsymbol{v} \tag{9.2}$$

Equations of Motion on disturbances take form:

$$AL\frac{\partial v}{\partial t} = \beta_2\nabla\, ra(\boldsymbol{z}) \quad ; \quad RA\frac{\partial u}{\partial t} = \beta_1\nabla\, al(\boldsymbol{z}) \tag{9.3}$$

Eq.(9.1-9.3) allow derive equations on *al* and *ra*

$$\left[\frac{\partial^4}{\partial t^4} - a\Delta\,\frac{\partial^2}{\partial t^2} + b\Delta^2\right]al(t,\boldsymbol{z}) = 0 \tag{9.4}$$

$$a = \alpha_1\beta_2 + \alpha_2\beta_1 \;;\; b = \beta_1\beta_2(\alpha_1\alpha_2 - 1)$$

Derivation of (9.4) from equations (9.2-9.3) is simple and we omit it here. For

$$c_{1,2}^2 = \frac{a+/-\sqrt{a^2-4b}}{2} > 0$$

(9.4) takes form of bi-wave equations:

$$(\frac{\partial^2}{\partial t^2} - c_1^2\Delta)\,(\frac{\partial^2}{\partial t^2} - c_2^2\Delta)al(t,\boldsymbol{z}) = 0 \tag{9.5}$$

Here $c_{1,2}$ - different velocities of field disturbances waves propagating on e-space. Green function of bi-wave equation (9.5) equals convolution of Green functions of common wave equations with wave speeds equal c_1 and c_2. Thus even simple δ-function shocks induce complex wave response. Equations (9.4) or (9.5) validate diversity of wave processes that govern evolution of macro transactions. Thus field disturbances can induce waves that propagate through e-space domain and may cause time fluctuations of macro variables as Assets, Investments, Profits, Capital, etc. Let show that equations (9.4) admit wave solutions with amplitudes growth up as exponent in time. Let take $al(t,\boldsymbol{z})$ as:

$$al(t,\boldsymbol{z}) = \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{z})\exp(\gamma t) \ \ ; \quad \boldsymbol{k} = (\boldsymbol{k}_x, \boldsymbol{k}_x) \tag{10.1}$$

Solution (10.1) satisfies equations (9.4) if:

$$\omega^2 = \gamma^2 + \frac{ak^2}{2} \qquad 4\gamma^2\omega^2 = k^4\left(b - \frac{a^2}{4}\right) > 0 \ ; \ 4b > a^2$$

$$\gamma^2 = k^2\frac{\sqrt{4b+3a^2}-2a}{8} > 0 \qquad \omega^2 = k^2\frac{\sqrt{4b+3a^2}+2a}{8} > 0$$

For $\gamma > 0$ wave amplitudes grow up as $exp(\gamma t)$. Relations (10.1) describe simple harmonic waves of Assets-Liabilities field disturbances $al(t,\boldsymbol{z})$ with amplitudes growing up in time as exponent. Due to definition of e-space in Section 2 coordinates of e-particles define their risk ratings. Thus, for simplest 1-dimensional e-space R Assets-Liabilities field $AL(t,\boldsymbol{z}=(x,y))$ is determined on e-space R^2. Let assume that risk ratings of e-particles are reduced by minimum X_{min} and maximum X_{max} risk grades. For simplicity let take borders of e-space domain as $X_{min}=0$ and $X_{max}= X$. Hence on e-space

$$0 \le x \le X \tag{10.2}$$

Due to (9.1) Assets-Liabilities field $AL(t,\boldsymbol{z}=(x,y))$ is presented as

$$AL(t,\boldsymbol{z}) = AL + al(t,\boldsymbol{z}) \tag{10.3}$$

For assumption (10.1-10.3) rate of Assets change $A(t)$ at moment t equals

$$A(t) = A_0 + a(t) \ ; \ A_0 \sim AL\,X^2$$

$$\mathrm{a(t)} = \frac{4\exp(\gamma t)}{k_x k_y}\cos\left(\frac{k_x+k_y}{2}X - \omega t\right)\sin\frac{k_x}{2}X\sin\frac{k_y}{2}X$$

Hence rate of total Assets $A(t)$ growth follows time oscillations with frequency ω. For $\gamma>0$ amplitude of Assets growth fluctuations may increase in time as $exp(\gamma t)$. For $\gamma<0$ amplitude of Assets growth dissipate and tend to constant rate A_0. These examples illustrate relations between time oscillations of rate of growth of macro Investment on one hand and simple model of interactions between Assets-Liabilities and Revenue-on-Assets macro transactions and their disturbances waves on e-space on the other hand. Thus we show that relations between macro variables like Investment, Assets, Credits and etc., treated as functions of time can be determined by complex interactions between *conjugate* macro transactions as functions of time and coordinates on e-space R^{2n}. Macro financial variables at point $\boldsymbol{x}$ are determined by complex interactions of transactions between agents with risk ratings $\boldsymbol{x}$ and $\boldsymbol{y}$ on e-space. Equations on financial field disturbances admit wave solutions and can describe exponential growth of wave amplitudes in time.

Conclusions

Any theory is based on certain assumptions. We present a macro financial model in assumption that it is possible develop econometrics and risk assessments of economic agents in a way required for modeling macro finance on economic space. We assume that risk assessment methodology can be extended in such a way that risks ratings for huge banks and corporations, small companies, householders and personal investors can be estimated and measured. We propose that econometrics can select "independent" agents and measure financial variables of all agents under consideration. Of course it can't be done precisely for each economic agent and probability distributions should be used to define values of financial variables of agents. We suppose that econometrics can help measure financial transactions between agents on economic space and that is additional and extremely tough problem.

Economic space notion is a core issue of our approach to macro finance. Introduction of economic space as generalization of agent's risk ratings permit describes agents by their coordinates on economic space. Nature of macro finance system is completely different from physical systems but certain similarities between them allow develop models *alike to* kinetics and hydrodynamics. Economic space is determined by risk grades of most valuable risks and has different representations for different set or major risks. Random properties of risk nature cause random changes of economic space representation. There are no ways to establish determined macro financial forecast as random nature of risks growth and decline insert permanent uncertainty into macro dynamics and modeling on economic space. Possibility to measure and select most valuable financial risks should establish procedure to validate the initial and target set of risks and to prove or disprove initial model assumptions. It makes possible to compare predictions of financial models with observations and helps outline causes of disagreement between theoretical predictions and macro financial reality. Development of sufficient econometric ground requires collective efforts of Financial and Economic Regulators, Rating Agencies and Market Authorities, Businesses and Government Statistical Bureaus, Academic and Business Researchers, etc. Achievements in developments of national accounts (Fox et al., 2014) and Leontief's input-output inter-industry tales (Horowitz and Planting, 2009; Miller and Blair, 2009) prove that such problem can be solved.

We regard transactions between agents as principal tool for implementation of financial processes. Transactions between agents change agent's financial variables and hence induce changes of corresponding macro variables. Aggregation of amount of transactions between agents at points $\boldsymbol{x}$ and $\boldsymbol{y}$ define macro transactions between points $\boldsymbol{x}$ and $\boldsymbol{y}$ on economic space. We model macro transactions by hydrodynamic-like equations on economic space determined by coordinates $(\boldsymbol{x},\boldsymbol{y})$. Evolution of macro transactions defines dynamics of macro financial variables and thus describes dynamics and state of macro financial system. As example for simple model interaction between Assets-Liabilities and Revenue-on-Assets we obtain hydrodynamic-like equations in a closed form. That permits describe evolution of such important macro variables as Asset, Liabilities, Revenue-on-Assets. For this model we derive financial wave equations on disturbances of macro transactions. We show that disturbances and shocks of transactions between agents induce financial waves that propagate on economic space from high to low risk ratings area or vise versa. Wave propagation of shocks of transactions induces fluctuations of financial variables and should be important for further modelling of financial fluctuations. Influence of financial wave processes on macro finance evolution can explain and describe development of crises, propagation of instabilities, business cycles and etc.

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