HOW MAKE REAL OPTION DEPENDABLE AND UNDERSTANDABLE AS A STRATEGIC DECISION MAKING TOOL?

ZAHRA SHAHROKHSHAHI¹

¹Universidad Carlos III de Madrid, Finance Department

Abstract: Real option methodology has gained surge of attention by scholars after its introduction, yet in real world application, in spite of general acceptance of the model, decision makers do not trust the model extensively. Focusing on different criticisms of the model, this paper is a try to make real option model more trustworthy for real world application. At first, the topic of capturing all the concurrent options investigated with assumption of mean reversion stochastic process and dynamic leakage rate for deferment in start a project. Then, in next step, distribution based model introduced to face the issue of option pricing with two sources of risk, market and private risks. The new model treat different risks individually, besides, its result is suitable for strategic decision making process. As a response to the necessity of proper understanding of real option result, its interpretation mentioned ,briefly, by using Pascal triangular. Finally the result of proposed approaches reviewed applying a real world case.

Keywords: Defer option, Private risk, Multiplex model, distribution based model, real option interpretation

Introduction

Capital investment decision-making is an important challenge for managers. Economic valuation of projects and making strategic decision about their initiation or continuation is a complex task, which take into consideration significant sums of money to invest, long lifetime of a project, and project return and market uncertainties. Therefore, it is crucial for an organization to accurately measure the benefits and risks of project during the planning phase.

Real option methodology, introduced in 1990s, is an advanced method to evaluate the economic value of a project, inspiring from financial option tool. For the last two decades, real option approach has gained a surge of attentions by scholars and has an established academic tradition, yet, some surveys show that the method is not widely in use by managers because of the gap between real world characteristics and real option model. Also, there is lack of insight about the interpretation of the result (H. Kent Baker, Shantanu Dutta, Samir Saadi , 2011), (Block, 2007).

Motivated by the aforementioned problem, and to add to this line of research, this paper will focus on two improvement to the model and try to clarify the real option result interpretation.

The added value in real option model ensued from the fact that decision makers have authority to change the project path. Based on the project characteristics, decision maker would have option to defer, expand, abandon, contract and switch the project. Although all projects may not have all the kind of flexibilities at the same time, yet a good model should be able to capture all of these

aforementioned options. Also, we should consider the fact that deferment to start a project, in most cases, would cause dynamic depreciation of income, or increase in expenses. Then the first part of the paper is a try to find a real option model which capture all type of manager's flexibilities, including defer option, with assumption of dynamic leakage rate.

The second challenge of this paper is suggestion of a real option model with consideration of private and market risks. Due to intrinsic difference between these two sources of risk, they have distinct effect on real option value. Then a reliable model should be able to encounter them individually. Besides, the final result should be detachable based on private risk and market risk, if we believe real option model as a strategic decision making tool.

Finally, real option is a contributory model if decision maker be clear about interpretation of its result. To enhance the interpretation of real option model result, it is tried to be addressed as an strategic decision making tool. Then, the model applied through a case study to analyze the result.

The remainder of this article is organized as follows. In next section, a brief description of our assumptions and related literature bring up. Suggested multiplex real option model as a more comprehensive model discussed in section 3. Section 4 is an attempt to integrate private risk and market risk in real option pricing model. The topic of real option interpretation studied in detail in section 5. Section 6 presents a real case application of the proposed model and provides sensitivity analysis of the result. Finally, Section 7 draws the conclusions of the research.

Literature Review

In real option model, the same as financial options, there are various pricing method. Compared with other option pricing methods, binomial method has higher flexibility in modeling multiple uncertainties, multiple concurrent options and complex payoff characteristics. Therefore, considering the fact that options involved in real projects are more complicated than financial options, binomial tree is a more suitable evaluation method for real option applications (Z.Zmeškal, 2010). In addition to its computational advantages, binomial tree model provides an efficient discrete approximation of stochastic process, which is a transparent answer for manager to analyze (W.J.Hahn; J.S.Dyer, 2008).

Also, all the discussions in this paper is based on presumption that commodity prices follow from mean reversion process. Indeed, in literature, it is commonly accepted that mean reversion process gives better estimation of commodity prices, due to long life of commodities versus financial assets and supply-demand effect on commodities (E.S.Schwartz, 1997). Therefore the focus of this article is on binomial tree of Nelson-Ramaswamy model (D.B.Nelson, K.Ramaswamy, 1990) and mean-reversion binomial option pricing model as base models for real option pricing (W.J.Hahn; J.S.Dyer, 2008).

Defer option -the timing flexibility of the manager to start a project- is one of the most studied flexibilities in real world projects. Although deferment of a project would cause getting more information, it would cost losing the share of market and supplementary investment as well. Leakage rate is introduced ,in real option literature, to capture this extra expenditures (P.Kodukula, P.C.Papudesu, 2006) which is evaluated the same as dividend in option pricing models. Current models evaluate defer option for commodities with deterministic leakage rate and Brownian motion. While the focus of this paper is a model of defer option pricing with mean reversion process and dynamic leakage rate.

With the aim of capturing all concurrent flexibilities in a project, except for defer option, choose option model had been introduced (Benaroch, 2002). The extension of Benaroch model which is able to capture all the concurrent flexibilities including defer option, suggested by Özogul (C.O.Özogul, E.E.Karsak, 2009). Although the new model capture all the manager's flexibilities in a project, yet need to be extended to a model with leakage rate for defer option, and mean reversion process for future prices like real world.

To have reliable real option model, in addition the necessity to capture all kind of manager's flexibilities, we should pay attention to different source of risks, market risk and private risk. Amram explain this concept in his promising book (M.Amram, N.Kulatilaka, 1999). Borison emphasized that real option model should not face with these two sources of risk in the same manner (Borison, 2005). There are several papers which try to find a suitable model to encounter private and market risks individually, in project valuation. The first attempts in this regards was the integrated model of decision making tree with real option model (J.E. Smith, R.F. Nau, 1995). Several endeavor done to improve the integrated model, because the method suffered from lack of flexibility to capture different concurrent options. For instance, Brando applied dynamic programming to solve the integrated model as one of the most developed integrated models (J.S. Dyer, W.J. Hahn, 2005). In spite of new approach suggestions, these models still suffer from lack of flexibility to evaluate more complicated options. Besides, their final result is not differentiable to the various effect of market and private risks.

Another major approach to the problem is applying risk adjusted value, instead of risk neutral value, to seize the effect of private risk. The first critics of these type of models was the inappropriateness of a subjective value. As a solution to the problem, Arnold suggested substitute of WACC (weighted average cost of capital) for risk adjusted value (T.Arnold, T.F.Crack, 2004). This measure increase the adjusted-risk and then option value (call option), while private risk in definition contains only the downside of uncertainty and consequently would decrease the option value.

Inspired from risk-adjusted idea, utility function employed to capture the unhedgeable part of the risk which is well reviewed in the paper of Matter (M.H.Mattar, 2006) .The idea extended further by several authors, for instance look at (M.R.Grasselli, 2011). We believe that although, manger's preferences should be considered in decision making process, yet it would not cover the private risk in a project. As an answer to the need of a model to combine private and market risks properly, In this paper, a simple integrated approach is suggested.

Finally, we will address the real option interpretation value. Real option methodology is not only an advanced economic evaluation model with a deterministic value as a result, but it is capable to be applied as a strategic decision making tool (D.N.Forda , D.M.Landerb, 2011). However, in real option literature, there are several studies that criticize the difficulty of understanding and interpretation of the real option result as the main reason for its lack of application (T.Copeland, Antikarov, 2005), (S.Block, 2007), (H.K.Baker, S.Dutta, 2011). In spite the necessity of studying real option result, it is not concentrated in literature properly. In context of real option result interpretation, study of Pascal triangular briefly mentioned by Kodukula and Papudesu (P.Kodukula, C.Papudesu, 2006). To make the idea more intuitive, the Pascal triangle interpretation method will be exemplified and discussed for better insight.

As previously mentioned, there are different criticisms on real option model which stem from the fact that real option model is not fit to real world characteristics. As an answer to this necessity, In following sections we will discuss and suggest models to address these criticisms to find a more reliable real option model.

Multiplex Real Option Model

Decision making about the initiating time of a project is a momentous subject for managers which is evaluated in real option models by defer option. The idea behind defer option is that later entrance in the market will probably cause getting more information and losing market share. Then the value in defer option model decrease by a leakage rate percentage for each period of deferring to enter the market. Defer option evaluation is exactly the same as option pricing with dividend, but it is not attempted in mean reversion models yet. In addition, in real world, the expense of delay in entering the market increases by time _dynamic leakage rate_, which is not addressed in literature. To develop the multiplex mean reversion option pricing model, at first we need to find a solution for mean reversion option pricing model with dividend. Then by applying the result to a defer option with leakage rate model, we will extend the multiplex real option model.

Mean reversion defer option

Defer option is similar to financial call option with dividend payment. The leakage rate in defer option decrease the value of the option per period, the same as dividend payment. With the aim of finding pricing model for defer option with leakage rate and mean reversion process, in following, we will try to derive mean reversion option pricing with dividend model. To find the desired result, we focused on two well known model of Nelson-Ramaswamy binomial tree.

Brownian motion binomial tree with dividend

Nelson_Ramaswamy (D.B.Nelson, K.Ramaswamy, 1990) proposed a general method for binomial option pricing model with fix up and down moves in each steps of the binomial tree (constant variance) and calculate the conditional probability of up moves, conditioned on the position of node to reflect the drift. The possibility of applying different kind of stochastic process in model, make it a general model. This model calculated based on Brownian motion (BM) stochastic process. In the BM process, the differential equation is equal to:

$$dS_t = r \, dt + \sigma \, dw_t$$

 $up = S_t + \sigma \sqrt{\Delta t}$

 $down = S_t - \sigma \sqrt{\Delta t}$

The up probability in this model is calculated with the similar idea of famous Cox-Ross-Rubenstein model (J.Cox, 1979), by using a substitute portfolio of stocks and bonds by the following equation as its basis.

$$P = \frac{S_t \left(1+r\right) - down}{up - down} \tag{*}$$

Applying the * equation for up probability value, with elementary calculation, Nelson_Ramaswamy derived the following values for the up probability in binomial tree.

$$P = \begin{cases} \frac{1}{2} + \frac{r\sqrt{\Delta t}}{2\sigma} & \text{if } 0 \le P \le 1\\ 0 & \text{if } P \le 0\\ 1 & \text{if } P \ge 1 \end{cases}$$

By adding the dividend rate (d) to the basic stochastic process equation, we reach the subsequent equation.

$$dS_t = (r - d) dt + \sigma dw_t$$

We can conclude the following up probability for BM binomial model with dividend from employing the related stochastic process equation and * equation.

$$P = \begin{cases} \frac{1}{2} + \frac{(r-d)\sqrt{\Delta t}}{2\sigma} & \text{if } 0 \le P \le 1\\ 0 & \text{if } P \le 0\\ 1 & \text{if } P \ge 1 \end{cases}$$

Mean reversion binomial tree with dividend

Mean reversion binomial tree option pricing model, for the first time, suggested by Nelson_Ramaswamy (D.B.Nelson, K.Ramaswamy, 1990). This model is based on the Ornestein-Uhlenbeck stochastic process (mean reversion process) with following differential equation $dS_t = \lambda (S_t - \bar{S}) dt + \sigma dw_t$

Where λ is the mean reversion coefficient, and \overline{S} is the long term value of S (underlined asset). This model is an extension of the BM model. The up and down movement, and up probability are respectively as follow:

$$up = S_t + \sigma \sqrt{\Delta t}$$

$$down = S_t - \sigma \sqrt{\Delta t}$$

$$P = \begin{cases} \frac{1}{2} + \frac{\lambda (S_t - \bar{S}) \sqrt{\Delta t}}{2 \sigma} & \text{if } 0 \le P \le 1 \\ 0 & \text{if } P \le 0 \\ 1 & \text{if } P \ge 1 \end{cases}$$

In financial mathematic contexts the differential equation of mean reversion process _i.e, Ornestein-Uhlenbeck _ with dividend is as follow

$$dS_t = \lambda \left[(S_t - \bar{S}) - d \right] dt + \sigma dw_t$$

Substituting the mean reversion drift for mean reversion drift with dividend in the Nelson-Ramaswamy model and applying the * equation, the up probability movement would be as follow:

$$P = \begin{cases} \frac{1}{2} + \frac{\lambda \left[(S_l - \bar{S}) - d \right] \sqrt{\Delta t}}{2\sigma} & \text{if } 0 \le P \le 1\\ 0 & \text{if } P \le 0\\ 1 & \text{if } P \ge 1 \end{cases}$$
(**)

Multiplex binomial tree with mean reversion process

Özogul and Karasak (2009) introduced a new binomial model which is able to capture all the manager's flexibilities (inclusding choose option and defer option). In spite of the advanteges of the model, it suffer from unrealistic presumptions, 1. Commodities follow from GBM process, 2. There is no leakage rate for deferment of the project. To develop the model to a more realistic one, in previous section we find the answer of mean reversion option pricing with dividend. To fill the gap of the model and reality, we apply the developed option pricing model in Özogul binomial tree. The eventuated model _multiplex binomial tree_ assume that prices follow from mean reversion process and there is a dynimc leakage rate for decision to delayed market entrance.

Multiplex binomial tree is combination of two groups of binomial trees, main binomial tree and several binomial sub-trees related to each node of the main tree, as presented in figure 1.In this model, the main tree (blue tree) evaluate the defer option and sub-trees (red trees) are responsible for calculation of choose option after making decision to enter the market. The process of pricing explained in details by figure 1.



Figure 1: Mean reversion twofold binomial tree

The first decision to make is the initial time of the project. In this model, we assume that every node in a binomial tree is a possible point to start the project, then evaluate the project initiating from each node as explained following in detail.

Each node in the blue tree represent a possible state in time, to decide to start the project or delay the investment. The initial value at the tree is the asset value (S₀) which produces up and down values by $S_t \pm \sigma \sqrt{\Delta t}$. This process continue to shape the complete binomial tree (S_{i,j}). In this model, at each node, separately, we calculate the value of continuing project suppose that it start at that point. To make the idea more clear, for example, consider the (2,1) node.

At this node we have the asset value $(S_{1,2})$ which will grow to up and down values of the red tree until shape the complete tree (project life time). Tree (2,1) evaluate the project value with option to choose by applying the ** equation. Although it should be noticed that the dividend value in this step would be equal to zero. The final value of the tree equals to option value plus the initial value which named as $F_{2,1}$. Applying the same calculation for all nodes of the blue tree, the final tree has $F_{i,j}$ values in each node which is a substitute for $S_{i,j}$ values.

The next step is to calculate the defer option in the blue tree. Pricing defer option in this step is the same as simple option pricing with dividend and up probabilities using the equation **.

Private Risk Distribution Based Model

Capturing market risk is the great advantage of real option model, yet a reliable model should be able to consider private risk as well. Private risk or operational risk is any source of risk other than market risk which include technology risk, ecological risk and so on. To attain the desirable model the following point should be considered.

1. The model be able to deal with market and private risk individually

In literature we always face with market risk and private risk terms, while they have distinct characteristics. Market uncertainty is possible to be hedged, then the decision maker could benefit from the upside changes and hedge its downside changes. In contrast, private risk is not possible to be hedged and will decrease the value of project. Then, it can be concluded that market risk (volatility) increase, while private risk decrease the option and consequently project value.

2. The result be separatable to the effect of market and private risk

In real option study, often, emphasis is on finding an accurate result, yet paid less attention to application of the result for decision making process. Real option result could be applied as an strategic decision making tool instead of a determinative entrance limit value. In this respect, a detachable result of real option model not only help managers to decide about a project based on their utilities, but also it will help them to design their strategic plan for the future.

The suggested model of private risk distribution, benefits from the aforementioned advantages. Besides, this approach is helpful to not lose the more detailed information which is usually available for private risk estimation.

In this approach, it is assumed that private risk affect the project during its life, and this event effect the initial value of asset _Net Present Value (NPV)_ directly. By applying various asset

values (NPVs), based on different circumstances of private risk, in real option model, the final result of the model is a distribution of outcomes with their related probabilities which is the more detailed result. Although, expected value of real option distribution could be calculated in a case of need for single value.

To make the idea more intuitive, for example, consider a pharmaceutical product which after 2 years of sample running, is planned to start market size production. At that time, besides a jump in revenue, the company encounter with chance of being, fully successful for 50%, semi-successful for 20%, and failed for 30%. Then there are three different NPVs, with their related probability for each possible outcomes. Applying the distribution based model, the final result is three different real option values, for each event, with related probabilities. Extending the example to a continues distribution of probable outcomes of private risk, the result would be an estimation of a distribution of real option values.

Figure 2 shows explicitly the option valuation process of private risk distribution model. Figure (2a) shows the distribution of private risk which could be approximated with discrete numbers with related probability of event. Each event of private risk, independent of the time of its happening, would cause a distinct NPV for the project which apply individually to the real option model (2b). Collecting real option values with regards to their related probability, result in a real option value distribution (2c). The star point in the figure picture a sample point process in model.



Figure 2: 2a, private risk distribution, 2b, real option binomial tree, 2c, real option value distribution. In these figures, star points show the process of values in model.

In spite of aforementioned advanteges of suggested approach, it increases the volume of calculation. Further, the volume of calculation increase exponentially with increment in the number of private risk sources. Then it is recommended that the model apply in cases considering their special characteristics.

Real option value interpretation

As regards real option model is a strategic decision making tool, it is not proper to use its result just as a single value, rather it is required to construe the result appropriately. As mentioned earlier, Pascal triangular approach gives a good intuition of real option interpretation (P.Kodukula, C.Papudesu, 2006). The following example presented to make the idea more intuitive. Consider a

project with option to expand, option to contract and option to abandon with initial value of 200 \$ and option value of 27\$. In figure 3 rectangular numbers show nodes with decision to expand project, oval numbers show nodes with decision to contract and lined number displays decision to abandon.





As shown in figure, each number of final nodes in Pascal triangular shows the number of paths end in the node (3b) which gives the probability of ending in each final node in a tree. Linking these probability calculation idea to the same size binomial tree, the probability of various possible future decisions for project could be calculated. In this example, the probabilities of each option exercising are as follow:

The number of possible paths after 5 years is 32 Probability of ending with expand decision: (1+5+10)/32=50%Probability of ending with contract option: (10+5)/32=46.9%Probability of abandoning the project: 1/32 = 3.1%

Although real option value is an important criterion for decision making of a project, but a single value is not all information that could be achieved from real option model. Studying the probability of future strategies, brings good understanding of the project value distinctively in two themes.

1: Required future investment: the required future investment is equal to the expected future cash flow, which results from exercising options. This cash flow could be positive from exercising abandon option or contract option or be negative due to payment of expand option exercise price. Indeed, the information of expected future investment helps managers to construct their portfolio of investments and design the company's strategic future plan.

2: Stability of project: The final decision of investment, depend greatly on the utility of decision maker. Applying the Pascal probabilities in real option model, brings good strategic information

about the stability of project and help managers to decide based on their priorities. The stability term here means the chance of continuing project without change or with favorable changes. As an instance, one's priority could be a project with higher real option value (economic estimated value), while other may prefer to have a project with stable situation with the aim of staying in the same line of business. Besides, we should be aware that any changes in project have intangible expenses that are not ,at least easily, possible to be captured by project evaluation methods, such as expense of managers inaction period, which make the stability study of the project more important.

To make discussion more understandable, consider the example of two different projects with the same possible future strategies _options_ and real option values. The first project have probabilities of 60% to continue project without change, 35% to expand project and 5% to abandon project, and the second project's probabilities are, 15% to continue project without change, 60% to expand project and 25% to abandon project. A risk seeking decision maker may choose the second project, due to its high probability of expansion, while a risk averse decision maker may choose the first project because of its trivial probability of abandon.

Forasmuch as mentioned, it is of importance that real option result do not be reported just as a single cutting value, and be studied well to help strategic decision making process. Besides of all the aforementioned discussion, it is also important to notice the effect of volatility and time to maturity on the option value. Increasing volatility cause increase the value of real option, while entering an uncertain market is not always favorable for managers, and depend on the decision maker's utility. Increase in time to maturity the same as volatility, increase the real option value. However, the long life of project afford more information about the project, yet, it is not always interesting for managers due to extra caused expenses.

Mean reversion multiplex real option in practice, analysis and interpretation

The suggested model applied in a mining project of deep sea rare metal extraction. In this special project managers flexibilities are recognized as option to expand, option to abandon, option to contract, and defer option.

The source of private risk in this project is the quality of extracted material which is provided by geological studies (figure 4). By the fact that this project is classified as a high technology project, the leakage rate is small value of 0.5% and grows, slowly, every year by 15%. In this study, the historical data of Osmium prices used as a substitute of the intended rare metals (figure 5). To estimate the volatility we use GARCH (1,1) model, and apply regression model for mean reversion coefficient assessment.



Figure 4: Probability distribution of private uncertainty (the quality of extraction)



Figure 5: Osmium daily prices information 2009-2015

To convert the continues values of private risk to discreet ones, several points with various probabilities from private risk distribution are chosen (Points are chosen per .02 difference in their probability of happening). In this model, each value of private risk results in an individual NPV and consequently real option value. Then the yield real option values with their related probability of happening utilized to approximate the continues distribution of real option value (figure 6). The expected value of mean reversion real option distribution is equal to 717,492 Mln \$, and the biggest probable real option value is 18646.15 Mln \$. To compare the result, we run the model for assets with GBM stochastic prices assumption. The expected value of GBM real option value is equal to 36630.29 Mln\$ which as it is expected, the real option value with mean reversion process is much less than the equal value with GBM process (figure 7).



Figure 6: Real option value distribution with mean reversion process



Figure 7: Real option value distribution with mean reversion and GBM process

Based on the Pascal triangular analysis, the probability of decision to expand the project is 96% and the probability of continuing project without change is 4%. The extreme result for expansion probability is due to trivial expansion exercise price and big expansion coefficient factor in this case. This result shows that the project is highly valuable and greatly stable in its future strategy, also it most likely means that manager should consider to start the project with its ultimate capacity. Although it is not possible to assess the priority of the project alone based on the above mentioned information.

In this case, the expected future investment is equal to 350 Mln (the expansion exercise price)* 96% (the probability of expansion) = 336 Mln which will occur at the first year. The sensitivity of the real option value based on changes of the exercise price presented in figure 8.



Figure 8: sensitivity of real option distribution value based on changes in expansion option exercise price (ke)

Considering that the decision in all the sub-trees is to expand project at the first node of the tree or continue project without exercising any of the available options, it is easy to conclude that the time to maturity did not add value to the final real option value. The same effect is observable for the volatility of prices, change in volatility value did not affect the real option value. Then, in this special case, the study of input parameters effect (volatility and time to maturity) is not beneficial.

Conclusion

The aim of this study is to develop real option model to a more trustable and understandable model for application. With this aim we tried to address tree important problems of real option model. At first, to approximate real world attitude with financial tools more realistic, we try to find a model which capture defer option as well as other manager's flexibilities with presumption that deferment of project would cost a dynamic leakage rate and asset prices follow from mean reversion process. The second focus to approach real world is to capture private risk as well as market risk. The suggested model treat with market and private risk separately and benefit from the advantages of the severable result based on different sources of risk. With regards to the importance of real option model result interpretation, the Pascal triangular technique explained and then utilized to allude to strategic decision making subject of corporate finance. Finally, the suggested model applied and analyzed in a real world case.

Although, the paper tried to address the topic of corporate finance decision making, yet, considering the great interest of topic for practical scientist, it requires to be studied more specifically. The future extension of the model would be focusing on the corporate finance decision making based on real option probabilities, especially extending a decision making model for portfolio of projects based on real option values.

References

- Benaroch, M. (2002). Managing Information Technology Investment Risk: A Real Options Perspective. *Journal of Management Information Systems*, 48-84.
- Block, S. (2007). Are "real option" actually used in the real world? Taylor and Francis, 255-267.
- Borison, A. (2005). Real options analysis: where are the emperor's clothes? Journal of applied corporate finance.
- C.O.Özogul, E.E.Karsak. (2009). A real options approach for evaluation and justification of a hospital information system. *journal of Systems and Software*, 2031-2102.
- D.B.Nelson, K.Ramaswamy. (1990). Simple binomial process as diffuision approximation in financial models. *Review* of *Financial Studies*, 393-430.
- D.N.Forda, D.M.Landerb. (2011). Real option perceptions among project managers. Risk Management, 122-146.
- E.S.Schwartz. (1997). The stochastic beehavior of commodity prices: Implications for valuation and hedging. *The journal of finance*, 923-973.
- H. Kent Baker, Shantanu Dutta, Samir Saadi . (2011). Corporate Finance Practices in Canada: Where Do We Stand? *Multinational Finance Journal*, 157-192.
- H.K.Baker, S.Dutta. (2011). Management views on real options in capital budgeting. *Journal of Applied Finance*, 18-29.
- J.Cox, S. M. (1979). Option pricing: A simplified appraoch. Journal of financial economics, 229-263.
- J.E. Smith, R.F. Nau. (1995). Valuing risky projects: option pricing theory and decision analysis. *Management science*, 795-816.
- J.S. Dyer, W.J. Hahn. (2005). Using Binomial Decision Trees to Solve Real-Option Valuation Problems. *Decision Analysis*, 69-88.
- M.Amram, N.Kulatilaka. (1999). *Real Options: Managing Strategic Investment in anUncertain World*. Boston, MA: Harvard Business School Press.
- M.H.Mattar, C. (2006). Valuing large engineering projects under uncertainty:private risk effects and real options. *Construction Management and Economics*, 847-860.
- M.R.Grasselli. (2011). Getting Real with Real Options: A Utility–Based Approach for Finite–Time Investment in Incomplete Markets. *Journal of Business Finance & Accounting*, 740-764.
- P.Kodukula, C.Papudesu. (2006). project valuation using Real option. florida: Ross Publishing, Inc.
- P.Kodukula, P.C.Papudesu. (2006). Project valuation using real option. Andrews way, Florida: Ross Publishing, Inc.
- S.Block. (2007). Are 'real options' actually used in the real world? *The engineering economist*, 255-267.
- S.Sarkar. (2003). The effect of mean reversion on investment under uncertainty. *Journal of Economic Dynamics and Control*, 377-396.
- T.Arnold, T.F.Crack. (2004). Using the WACC to Value Real Options. Financial Analysts Journal.
- T.Copeland, Antikarov. (2005). Meeting the Georgetown Challenge. Journal of applied corporate finance, 32-51.
- W.J.Hahn; J.S.Dyer. (2008). Discrete time modeling of mean-reverting stochastic processes for real option valuation. *European journal of operational research*, 534-548.
- Z.Zmeškal. (2010). Generalised soft binomial American real option pricing model. *European Journal of Operational Research*, 1096-1103.