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COMPARING THE PRECISION OF DIFFERENT METHODS OF ESTIMATING VAR WITH A FOCUS ON EVT

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Abstract: *The paper aims to conduct a comprehensive research in sphere of risk measurement. This study would like to determine the forecasting precision of different risk estimation tools through implication of popular methods e.g. parametric and non-parametric methods in this field and more fresh and complicated methods e.g. semi-parametric methods and finally confirming the results with exploiting backtesting methods. Design/methodology/approach – The paper opted for a quantitative approach of measuring VaR. Estimating VaR by implying 8 different methods then comparing the obtained results based on backtesting criterion. We put into examination 6 major international stock exchange indices e.g. Canadian TSX, French CAC40, German DAX, Japanese Nikkei, UK FTSE100 and US S&P500 from 03-June-2003 to 31-March-2014 meanwhile we used rolling-window technic for backtesting purpose. The data were obtained from Yahoo! Finance. Findings – The paper empirically determined extend to which, the aforementioned methods are reliable in estimating one-day ahead VaR. we find out that EVT and HS are the two most precise methods albeit at very high confidence levels the EVT produces the most accurate forecasts of extreme losses. Results of this study encouraged financial managers to turn from using traditional methods of risk measurement to more fresh and reliable one such as EVT method of estimating VaR. Originality/value – This paper fulfills need to a comprehensive study of different proposed methods of measuring risk and showed the estimated VaR of them in a readily comparative manner.*

Keywords: *VaR, HS, GARCH (1, 1), EGARCH, GJR-GARCH, AGARCH, DCC-MGARCH, FHS, EVT, Simulation Technique.*

Introduction

Notion of risk refers to a probability of happening some undesirable event, which is closely related to uncertainty. For financial risks, appropriate definition might be "any event or action that may adversely affect on organization's ability to achieve its objectives and execute its strategies". Indeed, two essential tasks of financial managers[*] are to a) forecast these adverse events and b) evaluate the market risk exposure by estimating losses -in advance – that is expected to occur in time of when the price of assets fall down. This is the purpose of the Value-at-Risk (VaR) methodology. VaR is a special type of "downside risk measure". The concept of VaR is

* Management of risk is briefly made up of the subsequent basic activities: a) understanding the risks being taken by an institution, b) measuring the risks, c) controlling the risk, d) communicating the risk."

easy albeit, its calculation is not. The methodologies initially developed to calculate VaR are: (a) Parametric method, (b) Non-parametric methods[†] and (c) Semi-parametric method. VaR not only produce a single statistic or express absolute certainty but also it makes a probabilistic estimate, and consequently refers to concept of randomness. Initially VaR ask, with taking into account a specific confidence level, what is our maximum expected loss over a specific time span?

Since VaR is the acknowledged method by the Basel Committee on Bank Supervision (BCBS)[‡], in result a growing body of literature has either proposed a new model for measuring VaR or compares the precision of VaR estimation by the competitive models. This paper contributes to comparison of several VaR using a comprehensive range of parametric, non-parametric and semi-parametric methods.

The assumption in modeling VaR e.g. normal distribution of return data series is not a realistic assumption in financial markets where the data series have thick tails, which are known by extreme events left outside the bounds of a normal distribution in modeling VaR. Neftci (2000) argues that it is likely that extreme events are “structurally” different from the return-generating process under market conditions. An obvious response to this problem is to employ a methodology that explicitly allows for the fat-tailed nature of return distributions, such as those based on Extreme Value Theory (EVT), which will be empirically examined in this paper.

Literature concerning the measure of volatility and the frequency of data to be used in parametric and non-parametric VaR is broad. Taylor and Xu (Taylor and Xu, 1997) and Anderson and Bollerslav (Andersen and Bolerslav, 1998) introduce the idea of realized volatility. ARCH family, which are considering as Parametric methods, was introduced by Engle (Engle, 1982) and GARCH introduced by Bollerslev (Bollerslev, 1986). First and most popular models allowing for asymmetrical impact of new information were: EGARCH (Nelson, 1991), TARCH (Zakoian, 1994) and GJR-GARCH (Glosten et al, 1993). Other model, most general from all presented in this dissertation is APARCH (Ding, Granger and Engle, 1993). Dave and Stahl (Dave and Stahl, 1997) showed the effects of ignoring volatility clustering and non-normality of daily returns distribution on VaR modeling. They provide a good review of VaR estimation techniques and their paper is already a classical source of reference. The idea of using intraday data when estimating volatility comes up in work of Merton (Merton, 1980). Boudoukht et al. (Boudoukht et al. 1997) applied a class of volatility models on comparison of interest rate volatility forecasts and concluded that” density estimation and Risk Metrics™ forecasts to be the most accurate for forecasting short-term interest rate volatility”.

Giovanni Barone-Adesi and Kostas Giannopoulos refinement the *Historical Simulation* (HS) methods and proposed the *Filtered HS* (FHS) in which based on results FHS outperform the HS in estimating Value-at-risk. Jacob Boudoukht et al. introduced HS (1998), which “avoids the parameterization problem entirely by letting the data dictate precisely the shape of the distribution”.

Risk managers and Portfolio managers concern extreme negative side movements in the financial markets. A long list of research has posted on this topic that is Semi-parametric technique. *Ramazan et. al.* (2006) examine the dynamics of extreme values of overnight borrowing rates in an inter-bank money market. *Generalized Pareto* distribution has been picked for it’s well fitting. *Fernandez* (2005) used extreme value theory to the United States, Europe, Asia, and Latin America financial markets for computing value at risk. One of his findings is on

† (a) and (b) are also known as “conventional methods”.

‡ BCBS involving the chairman of the central banks of Belgium, Italy, France, Swiss, Sweden, Spain, Holland, Canada, Luxemburg, Japan, the United States and the United Kingdom. This committee provides recommendations on banking regulations with regard to market, credit and operational risks. Its purpose is to ensure that financial institutions hold enough capital on account to meet obligations and absorb unexpected losses.

average, EVT provides the most accurate estimate of VaR. Byström (2005) applied extreme value theory to the case of extremely large electricity price changes and declared a good fit with generalized Pareto distribution (GPD). *Bali* (2003) determines the type of asymptotic distribution for the extreme changes in U.S. Treasury yields. *Neftci* (2000) found that the extreme distribution theory fit well for the extreme events in financial markets. *Gencay and Selcuk* (2004) investigate the extreme value theory to generate VaR estimates and study the tail forecasts of daily returns for stress testing. Marohn (2005) studies the tail index in the case of generalized order statistics, and declares the asymptotic properties of the Fréchet distribution. *Brooks, Clare, Dalle Molle and Persand, G.*, (2005) apply a number of different extreme value models for computing the value at risk of three LIFFE futures contracts. In this paper we will empirically estimate VaR based on EVT as well.

In the present paper, we perform an evaluation of the predictive performance of the conventional VaR methods e.g. non-parametric and parametric models as well as semi-parametric methods, which are initially mixture of the two previous methods. The models are "backtested" for their out-of-sample predictive ability by using Christoffersen's (1998) likelihood ratio tests for coverage probability. We put into examination 6 major international stock exchange indices e.g. Canadian TSX, French CAC40, German DAX, Japanese Nikkei, UK FTSE100 and US S&P500 from 03-June-2003 to 31-March-2014.we used rolling-window technic for back-testing purposes. The data were obtained from is Yahoo! Finance. The return series have been converted into logarithmic returns. Having homogeneous data of only mature capital markets, which due to their close relationship expect to have similar characteristics, was the main reason behind choosing the considered data series.

The study is organized as follows. In section 2, we review a full range of methodologies developed to estimate VaR. In section 2.1, a non-parametric approach is presented. Parametric approaches are offered in Section 2.2, and semi-parametric approaches in Section 2.3. In section 3, the obtained empirical results of comparing VaR methodologies are shown.

Theoretical characteristics of VaR models

Jorion (2001) said that under normal market condition and at a given level of confidence VaR is the worst expected loss over a certain horizon. For example, a financial institution might say that the daily value-at-risk of its trading stock position is \$1 million at the 95% confidence level. It means, under normal market conditions, only 5% of the time, the daily loss will beat \$1 million. In fact the value-at-risk just point out the most we can expect to lose if no negative event occurs.

Therefore value-at-risk is a conditional quantile of the asset return loss distribution. Based on *Jorion* (1990, 1997) "among the main advantages of VaR are simplicity, wide applicability and universality". Let $r_1, r_2, r_3, \ldots, r_n$ be $i.i.d.$ random variables representing the financial returns. Use $F(r) = P_r(r < r|\Phi_{t-1})$ conditionally on the information set Φ_{t-1} that is available at time $t-1$. Assume that $\{r_t\}$ follows the stochastic process:

$$r_t = \mu + \varepsilon_t \, ; \varepsilon_t = z_t \sigma_t \quad z_t \sim iid\,(0,1) \tag{1}$$

where $\sigma_t^2 = E(z_t^2|\Phi_{t-1})$ and z_t has the conditional distribution function $G(z), G(z) = \Pr(z_t < z|\Phi_{t-1})$. The value-at-Risk with a given probability $q \in (0,1)$, denoted by VaR (q), is described as the q quantile of the probability distribution of financial returns: $F\big(VaR(q)\big) = \Pr\big(r_t < VaR(q)\big) = q$ or $VaR(q) = \inf\{\upsilon | P(r_t \leq \upsilon) = q\}$.

This quantile can be valued in two ways: (a) inverting the distribution function of financial returns, $F(r)$, and (b) inverting the distribution function of white-noise[§], with regard to $C(z)$ the latter, it is also necessary to estimate σ_t^2.

$$VaR\ (q) = F^{-1}(q) = \mu + \sigma_t C^{-1}(q) \tag{2}$$

Hence, a value-at-risk model entails the specifications of function of innovations $C(z)$ or function of financial returns $F(r)$, we can carry out the calculation of these functions using the following methods: (1) Non-parametric methods; (2) Parametric methods and (3) Semi-parametric methods. Below we shall describe the methodologies, which have been developed in each of these three cases to estimate VaR[**].

Non-parametric Method

The major intend of Non-parametric approaches is to quantify an asset VaR without making strong assumptions about returns distribution. The core concept of these approaches is to "let data speak for themselves as much as possible" and not use to some assumed theoretical distribution rather recent returns empirical distribution- to estimate VaR. To be able to use the data from the recent past to forecast the risk in the near future all Non-parametric approaches are based on the underlying assumption that the near future will be satisfactorily similar to the recent past for us.

The Non-parametric approaches include (a) Historical Simulation (HS) and (b) Non-parametric density estimation methods. Since in this paper we empirically study VaR only based on Historical Simulation (HS), therefore, we will define properties of HS approach[††].

Historical simulation

In 1998 Historical Simulation (HS) was introduced in a series if paper by Boudoukh and *Barone-Adesi* as a method for estimating value-at-risk .HS is the most broadly applied Non-parametric and unconditional method. Research of *Perignon* and *Smith* (2010) recommend, "of the 64.9% of firms that disclosed their methodology, 73% (or three-quarter) reported the use of Historical Simulation rather than the parametric linear or MC value-at-risk methodologies". This model uses the empirical distribution of financial returns as an approximation for $F\ (r)$; hence VaR (q) is the q quantile of empirical distribution. Different sizes of samples can be taken into consideration to estimate the empirical distribution of financial returns. The keystone assumption is that the distribution of P&L is constant over the sample span and is a good predictor of future movements. In addition, this method is very sensitive to length of data sample as data may not be a good representative of current condition of market.

When value-at-risk is said as a percentage of the asset's value, the $100q\%\ n - day$ historical value-at-risk is the q quantile of an empirical $n - day$ discounted return distribution. The percentage value-at-risk can be transformed to value-at-risk in value terms: we just multiply it by the current portfolio value.

§ Also known as "innovations". Here we will use them interchangeably.

** For a more pedagogic review of some of these methodologies (see *Feria Dominguez,* 2005*)*.

†† For further studying about Non-parametric density estimation methods refer to *Bulter* and *Schachter* (1998) or *Rudemo* (1982).

Parametric methods (part of volatility models‡‡)

Parametric approaches calculate risk by firstly fitting probability curves to the data and next deducing the value-at-risk from the fitted curve. Among Parametric approaches, the first model to estimate VaR was RiskMetrics™ from *JPMorgan* (1996). This model assumes that the return portfolio add/or the residuals of return follow a normal distribution. Under this assumption, the value-at-risk of a portfolio at a $1-q\%$ confidence level is calculated as $\mathrm{VaR}\,(q)=\mu+\sigma_t C^{-1}(q)$, where $C^{-1}(q)$ is the q quantile of the standard normal distribution and σ_t is the conditional standard deviation of the return portfolio. To estimate σ_t, Morgan uses an Exponential Weight Moving Average Model (EWMA). The definition of this model is as follows:

$$\sigma_t^2(1-\lambda)\sum_{j=0}^{N-1}\lambda^i(\varepsilon_{t-i})^2 \tag{3}$$

where λ = 0.94 and the window size (N) is 74 days for daily data. Literatures have assigned a few drawbacks to the RiskMetrics that could be briefly listed as following:

- Normal distribution assumption for financial return and/or white-noises (see Bollerslev 1987).
- The model used EWMA to estimate the conditional volatility of the financial returns which it does not take into account symmetry and leverage effect (see Black 1976, Pagan and Schwert 1990)
- iid return assumption.

Given these disadvantages research on the Parametric methods has been made in several directions.

GARCH (1, 1)

In relate to the GARCH family, *Engle* (1982) proposed the “Autoregressive Conditional Heterocedasticity (ARCH), which features a variance that does not remain fixed but rather varies throughout a period”. *Bollerslev* (1986) further expended the model by including in the ARCH generalized model (GARCH). This approach identifies and calculates two equations: the first formulates the evolving volatility of returns, whilst the second sketch the evolution of returns in accord with earlier returns. The most generalized formulation for the GARCH models is the GARCH (p,q) model which is exemplified by the following statement:

$$r_t=\mu_t+\varepsilon_t \tag{4}$$

$$\sigma_t^2=\omega+\sum_{i=1}^{q}\alpha_i\varepsilon_{t-1}^2+\sum_{j=1}^{p}\beta_i\sigma_{t-j}^2 \tag{5}$$

Because initially GARCH model do not take into consideration the asymmetric performance of returns before positive or negative shocks (known as *leverage effect*) GARCH technique do not fully reflect the nature forced by the well-known properties of the financial time series, volatility. Meanwhile, they accurately characterize the volatility-clustering feature.

‡‡ the volatility models can be divided into three groups: (a) the GARCH fsmily (b) realized volatility-based models and (c) the stochastic models.

DCC-MGARCH model

Multivariate GARCH models, specified in *Engle* (2002), allow the conditional covariance matrix of the dependent variables to follow a flexible dynamic structure and allow the conditional mean to follow a vector autoregressive (VAR) structure. The general MGARCH model can be written as:

$$y_t = Cx_t + \varepsilon_t \qquad and \qquad \varepsilon_t = H_t^{1/2}\nu_t \tag{6}$$

where y_t us a m-vector of dependent variables, m is a $m \times k$ parameter matrix, x_t is a k-vector of explanatory variables, possibly including lags of $y_t, H_t^{1/2}$ is a *Cholesky* factor of the time-varying conditional covariance matrix H_t, and ν_t is a m-vector of zero-mean, unit-variance $i.i.d.$ Innovations[§§].

- EGARCH (1,1)

The Exponential GARCH (EGARCH) model assumes a specific parametric form for this conditional heteroskedasticity. More specifically, we say that $\varepsilon_t \sim$ EGARCH if we can write $\varepsilon_t = \sigma_t x_t$, where x_t is standard Gausian and:

$$\ln(\sigma_t^2) = \omega + \alpha(|x_{t-1}| - \mathbb{E}[|x_{t-1}|]) + \lambda x_{t-1} + \beta \ln(\sigma_{t-1}^2) \tag{7}$$

Besides leptokurtic returns, the EGARCH model, as the $GARCH$ model, captures other stylized facts in financial time series, like volatility clustering. The volatility is more likely to be high at time t if it was also high at time $t-1$. Another way of seeing this is noting that shock at time $t-1$ also impacts the variance at time t.

GJR-GARCH (1, 1)

The *Glosten-Jagannathan-Runkle* GARCH (GJR-GARCH) model assumes a specific parametric form for conditional heteroskedasticity. More specifically, we say that $\varepsilon_t \sim \mathrm{GJR-GARCH}$ if we can write $\varepsilon_t = \sigma_t x_t$, where x_t is standard Gaussian and:

$$\sigma_t^2 = \omega + (\alpha + \lambda I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{8}$$

where $I_{t-1} = \begin{cases} 0 \; if \; r_{t-1} \geq \mu \\ 1 \; if \; r_{t-1} < \mu \end{cases}$ (9)

besides leptokurtic returns, the GJR-GARCH model, like the GARCH model, captures other stylized facts in financial time series, like volatility clustering.

§§ A general MGARCH (1,1) model may be written as: $vech(H_t) = s + A\, vech(\varepsilon_{t-1}\varepsilon_{t-1}') + B\, vech(H_{t-1})$ where the $vech\,(.)$ function returns a vector containing the unique elements of its matrix argument. The various parameterizations of MGARCH provide alternative restrictions on H, the conditional covariance matrix, which must be positive definite for all t. Stata's $mgarch$ command estimates multivariate GARCH models, allowing both the conditional mean and conditional covariance matrix to be dynamic.

AGARCH (1,1)

AGARCH model besides leptokurtic returns captures other stylized facts in financial time series like volatility clustering.

Consider a return time series (1), where μ is the expected return and ε_t is a zero-mean white noise. Despite being serially uncorrelated, the series ε_t does not need to be serially independent. For instance, it can present conditional heteroskedasticity. The Asymmetric GARCH (AGARCH) model assumes a specific parametric form for this conditional heteroskedasticity. More specifically, we say that $\varepsilon_t\sim$ AGARCH if we can write $\varepsilon_t = \sigma_t z_t$, where z_t is a standard Gaussian and:

$$\sigma_t^2 = \omega + \alpha(\varepsilon_{t-1} - \lambda)^2 + \beta\sigma_{t-1}^2 \quad (10)$$

there is a stylized fact that the AGARCH model captures effects that is not contemplated by the GARCH model, which is the empirically observed fact that negative shocks at time $t-1$ have a stronger impact on the variance at time t than positive shocks. This asymmetry is called the leverage effect because the increase in risk was believed to come from the increased leverage induced by a negative shock.

Semi-parametric methods

The Semi-parametric methods concatenate the Non-parametric approach with the Parametric approach. The most important methods are Volatility-weighted Historical Simulation, Filtered Historical Simulation (FHS), CaViaR method and the method based on Extreme Value Theory.

In this paper we will probes properties of the first and the late method. Some application of Volatility-weighted Historical Simulation as well as *CaViaR* methods in VaR literature can be found in the following research papers: *Hull* and *White* (1998) and *Engle* and *Manganelli* (2004) respectively. *Hull et al*. indicates that this approach produces a VaR estimate superior to that if the Historical Simulation approach albeit, *Engle* et al. initially fails to provide accurate VaR estimate.

- Filtered Historical Simulation (FTS) with bootstrapping

Barone-Adesi (1999) introduced Filtered Historical Simulation (FHS) for fist time. This model combines the benefits of HS with the power and flexibility of conditional volatility models. FHS technique is an alternative to traditional *HS* technique and *Monte Carlo* (MC) simulation approach. Filtered *Historical Simulation* incorporates a nonparametric characteristic of the probability distribution of assets returns with a relatively complex model-based treatment of volatility (e.g. EGARCH). One of the interesting structures of *Filtered Historical Simulation* is its capability to produce reasonably large deviations (losses and gains) not found in the original asset return series. This method assumes that the distribution of returns of assets under examination is initially $i.i.d.$ To make the data $i.i.d.$ we must fit the first order autoregressive (AR1) model to the conditional mean of the asset returns, which can be formulized as:

$$r_t = c + \emptyset r_{t-1} + \varepsilon_t \quad (11)$$

and an asymmetric EGARCH model to the conditional variance

$$\log[\sigma_t^2] = \omega + qlog[\sigma_{t-1}^2] + \Phi(|z_{t-1}| - \mathbb{E}[|z_{t-1}|]) + \Psi z_{t-1} \quad (12)$$

the AR(1) model compensates for autocorrelation, whilst the Exponential GARCH model also combines asymmetry (leverage) into the variance equation (Nelson, 2005).

To compensate for the fat tails often related to index returns the standardized residuals of each index are modeled as a standardized Student's t distribution. That is

$$z_t = \frac{\varepsilon_t}{\sigma_t} \ \ i.i.d \text{ distributiont}(\upsilon) \tag{13}$$

Imagine we use FHS to estimate the value-at-risk of a financial asset over a 1-day horizon. The first step in applying this technique is to fit a conditional volatility model to the asset return data. *Barone-Adesi* et al. (1999) suggested an asymmetric GARCH model. The realized returns are then standardized by splitting each one by the corresponding volatility, $z_t = ({}^{\varepsilon_t}/_{\sigma_t})$. These standardized returns should be suitable for HS. The third step consists of bootstrapping a large number L of drawing from the above sample set of standardized returns.

Assuming a 1-day VaR horizon (or holding period), the third stage includes bootstrapping from our data set of standardized returns: we take a large number of drawings from this data set, which we now treat as a sample, substituting each one after it has been producing and multiplying each such random producing by the volatility forecast 1-day ahead:

$$r_t = \mu_t + z^* \sigma_{t+1} \tag{14}$$

where z^* is the simulated standardized return extracted from equation (13). If we take M producings, we therefore obtain a sample of M replicated returns. With this method, the VaR(q) is the $q\%$ quantile of the calculated return sample***.

Extreme Value Theory (EVT)

EVT approach concentrates on the limiting distribution of extreme returns observed over a long time span, which is indeed independent of the distribution of the returns themselves. The two main models for EVT are (a) the *Block Maxima model* (BM) (*McNeil*, 1998) and (b) the *Peaks Over Threshold* model (POT). In the POT method, there are two kinds of analysis: the Semi-parametric models built around the Hill estimator and its relatives (*Beirlant et al*., 1996; *Danielsson et al*., 1998) and the fully Parametric models based on the *Generalized Pareto Distribution* (GPD) (*Embrechts et al*., 1999). In this paper we apply POT with analysis type of GPD. In the coming sections each one of these approaches is described.

Detailed description of BM and the Semi-parametric models built around the *Hill estimator* can be found in *McNeil* (1998) and *Beirlant et al* (1996), respectively.

Peak Over Threshold model (POT). The POT model is initially said to be the most useful for practical applications because of more efficient use of the data for the extreme values. In this model, we can make a distinction between two types of analysis (a) the fully Parametric models based on the GPD and (b) the Semi-parametric models built around the Hill estimator. In this paper we shall merely introduce the first manner of analysis.

Firstly, in line with FHS method, we applied an EGARCH (1,1) model. The specific parameters of the model chose based on logarithmic returns, so residuals of the model will then become standardized and consequently with this technique we shall gain identically and independently distribution residuals.

*** To perform this analysis we used code of MATLAB *Statistic Tools.*

Secondly, the standardized identically and independently distributed residuals will be used to generate empirical *Cumulative Distribution* function based on *Gaussian kernel*. Based on general features of financial time series the kernel Cumulative Distributed Function estimation is expected to be well fitted to the interior of the distribution and performing poorly in lower and upper tail (this will be tested whether it will be correct or not for our dataset). For this reason we will implement extreme value theory for all observations that fall in each tail. We select thresholds levels e.g. 10 per cent of data belong to both right and left tail, and then fit the data that satisfy our condition (e.g. fall below defined threshold). This is also known as *peaks over thresholds* or *distribution of exceedances* method (Davison and Smith 1990).

Thirdly, we report value-at-risk of the considered indices at different confidence levels, from very low to very stringent intervals.

Generalized Pareto Distribution (GPD): Among the random variables demonstrating financial returns $(r_1, r_2, \dots, r_n)$, we pick a low threshold u and examine all values (x) exceeding u: $(x_1, x_2, \dots, x_{N_u})$, where $x_i = r_i - u$ and N_u are the number of sample data greater than u. The distribution of excess losses over the threshold u is defined as:

$$F_u(x) = P(r - u < x | r > u) = \frac{F(x+u) - F(u)}{1 - F(u)} \qquad (15)$$

Assuming that for a certain u, the distribution of excess losses above the threshold is a GPD, $G_{k,\xi}(x) = 1 - [1 + \left(\frac{k}{\xi}\right) x]^{-1/k}$ (28), the distribution function of returns is given by:

$$F(r) = F(x+u) = [1 - F(u)]G_{k,\sigma}(x) + F(u) \qquad (16)$$

To build a tail estimator from this statement, the only additional part we need is an calculation of $F(u)$. For this point, we take the evident empirical estimator $(u - N_u)/u$. Next we use the HS method. Presenting the historical estimate of $F(u)$ and setting $r = x + u$ in the equation, we arrive at tail estimator

$$F(r) = 1 - \frac{N_u}{n}[1 + \frac{k}{\xi}(r - u)]^{-1/k} \quad r > u \qquad (17)$$

For a given probability $q > F(u)$, the value-at-risk measure is calculated by inverting the tail estimation formula to obtain

$$VaR(q) = u + \frac{\xi}{k}[[\frac{n}{N_u}(1 - q)]^{-k} - 1] \qquad (18)$$

Where parameters ξ (shape parameter)and k (scale parameter) are estimated by MATLAB using Newton's method.

Backtesting VaR methodologies

This section presents applied *backtesting* methods to value-at-risk model validation across sample forecast evaluation methods. Failure of backtesting specifies that value-at-risk model misspecification and/or large estimation errors.

According to the endorsements of the Basel Capital Accord in 1996, we shall implement the "backtesting" technique to evaluate the reliability and precision of all model considered.

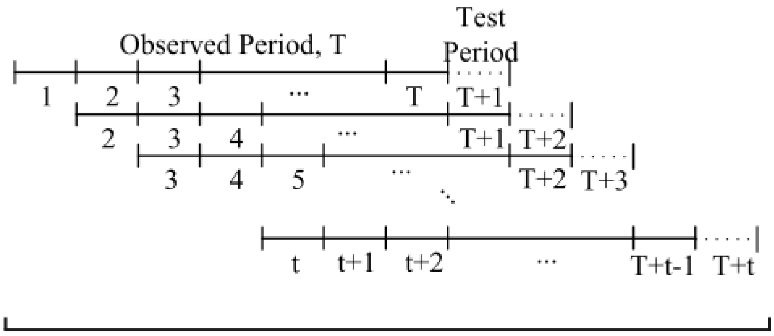

Figure. 1 Sliding window simulation process with estimation and test sample.

Source: Lin et al. (2009) page 2507 with slight modification by the author.

The general simulation process uses the *sliding window*[†††] methodology. One of the main benefits of this method is that we prevent *overlapping data* in the test sample. First, we determine an *estimation period*, which defines the sample used to calculate the value-at-risk model parameters. Then we employ sliding window approach as follows. The estimation period is progressively moves one time fracture until the end of our testing, keeping the calculating period the same, starting at the beginning of data span. Figure 1 clarifies the rolling window method: the dark grey line at the bottommost shows the entire sample covering the whole data period. The estimation and test samples are shown in grey and dotted line, respectively; during the backtest these are rolled gradually, n day at a time, until whole sample is ended.

In our testing, estimation sample size is 1500 and sample consists of 2774 daily observations and the risk horizon is one day ahead. The backtest proceeds as follows. Use the estimation sample to calculate the one-day VaR_q on the 1500 day. This is value-at-risk one-day return from the 1500th to the 1501th observation. Then, assuming the value-at-risk is stated as a percentage of the asset value, we observe the *realized return* over this one-day test period, and keep both the value-at-risk and the realized return. Then we slide the window forward one-day and iterate the prementioned process, until the entire sample is exhausted. The result of this procedure will be two time series covering the sample from 1501th until the 2774th observation. One series is the one-day value-at-risk and the other is the one-day "realized" return. The backtests is based on these two series.

The conventional tests about *the validity* of value-at-risk models are: (a) unconditional and conditional coverage tests; (b) the backtesting criterion and (c) the dynamic quantile test.

Most often backtests on daily value-at-risk are constructed on the assumption that the daily returns or $P\&L$ are generated by an identically normally independent *Bernoulli process.* A Bernoulli variable can take only two values, which could be labeled 0 and 1, or "failure" and "success". Thus we may define an *indicator function as* $I_{q,t}$ on the time series of daily returns or $P\&L$ relative to the q% daily VaR by

$$I_{q,t+1} = \begin{cases} 1, & if\ R_{t+1} < -VaR_{1,q,t}, \\ 0, & otherwise. \end{cases} \tag{19}$$

here R_{t+1} is the "realized" daily return or $P\&L$ on the portfolio from time t, when the value-at-risk estimate is made, to time $t+1$.

††† Another well-known phrase is "rolling window". In this paper we will use them interchangeably.

If the VaR model is accurate and $\{I_{q,t}\}$ follows an $i.i.d.$ Bernoulli process, the probability of "success" at any time t is q. Thus the nq is equal to expected number of success in a test sample with n observation. So we can use this information to build a two-sided 95% confidence interval for each of our indices

$$\left(nq - 1.96\sqrt{nq(1-q)}, nq + 1.96\sqrt{nq(1-q)}\ \right) \tag{20}$$

Kupiec (1995) shows that assuming the probability of an exception is constant, then the number of exceptions $x = \sum I_{t+1}$ follows a binomial distribution $B\ (N, q)$, where N is the number of observations. An accurate VaR (q) measure should produce an *unconditional coverage* ($\hat{q} = \sum I_{t+1}/N$) equal to q percent. The *unconditional coverage test* has a null hypothesis $\hat{q} = q$, with a likelihood ratio statistics:

$$LR_{UC} = 2[\log(\hat{q}^x(1{-}\hat{q})^{N-x}) - \log(q^x(1-q)^{N-x})] \tag{21}$$

Which follows an asymptotic $\chi^2(1)$ distribution.

Christoffersen (1998) developed a *conditional coverage test.* This jointly examines whether the percentage of exceptions is statistically equal to the one expected and the serial independence of I_{t+1}. He proposed an independence test, which aimed to reject VaR models with clustered violations. The likelihood ratio statistics of the conditional coverage test is $LR_{cc} = LR_{uc} + LR_{ind}$ (22), which is asymptotically distributed $\chi^2(2)$, and the LR_{ind} statistics is the likelihood ratio statistics for the hypothesis of serial independence against first-order Markov dependence.

$$LR_{ind} = \ -2\log[(1-\hat{q})^{(T_{00}+T_{10})}\left(\hat{q}\right)^{T_{01}+T_{11})}\big] + 2\log[(1-q_0)^{T_{00}}{q_0}^{T_{01}}(1-q_1)^{T_{10}}{q_1}^{T_{11}}] \tag{23}$$

Which follows an asymptotic $\chi^2(1)$ distribution.

VaR estimation and backtesting analysis

In this study, we implemented various methods of VaR estimation from all three main categories of VaR estimation techniques namely, Non-parametric methods (Historical Simulation), Parametric methods (GARCH (1,1), DCC-MGARCH, EGARCH, GJR-GARCH, and AGARCH (1,1)) and Semi-parametric methods (Filtered Historical Simulation with bootstrap and Extreme Value Theory).

The data used in estimation and forecasting are daily evolution of returns of 6 indices e.g. Canadian TSX, French CAC40, German DAX, Japanese Nikkei, UK FTSE100 and US S&P500, from 03-June-2003 to 31-March-2014. The index data were obtained from Yahoo Finance for the period June 3, 2003 to March 31, 2014. The computation of the index returns (r_t) is based on the formula $r_t = \ln(I_t/I_{t-1}) \times 100$, where I_t is the value of the stock-market index for period *t*.

Preliminary statistics for the data are presented in the inner box of figure 1. For all indexes, the unconditional mean of daily log-returns is close to zero. The maximum and minimum values are between −9.78% and 9.37% for the TSX index. The skewness statistics are negative for the Nikkei (−0.571), FTSE (−0.157), S&P (-0.336) and positive for the TSX (0.739) and CAC (0.040) and DAX (0.011) .For most indexes considered, these values are very close to zero, implying that the distributions of these returns are not far from symmetric.

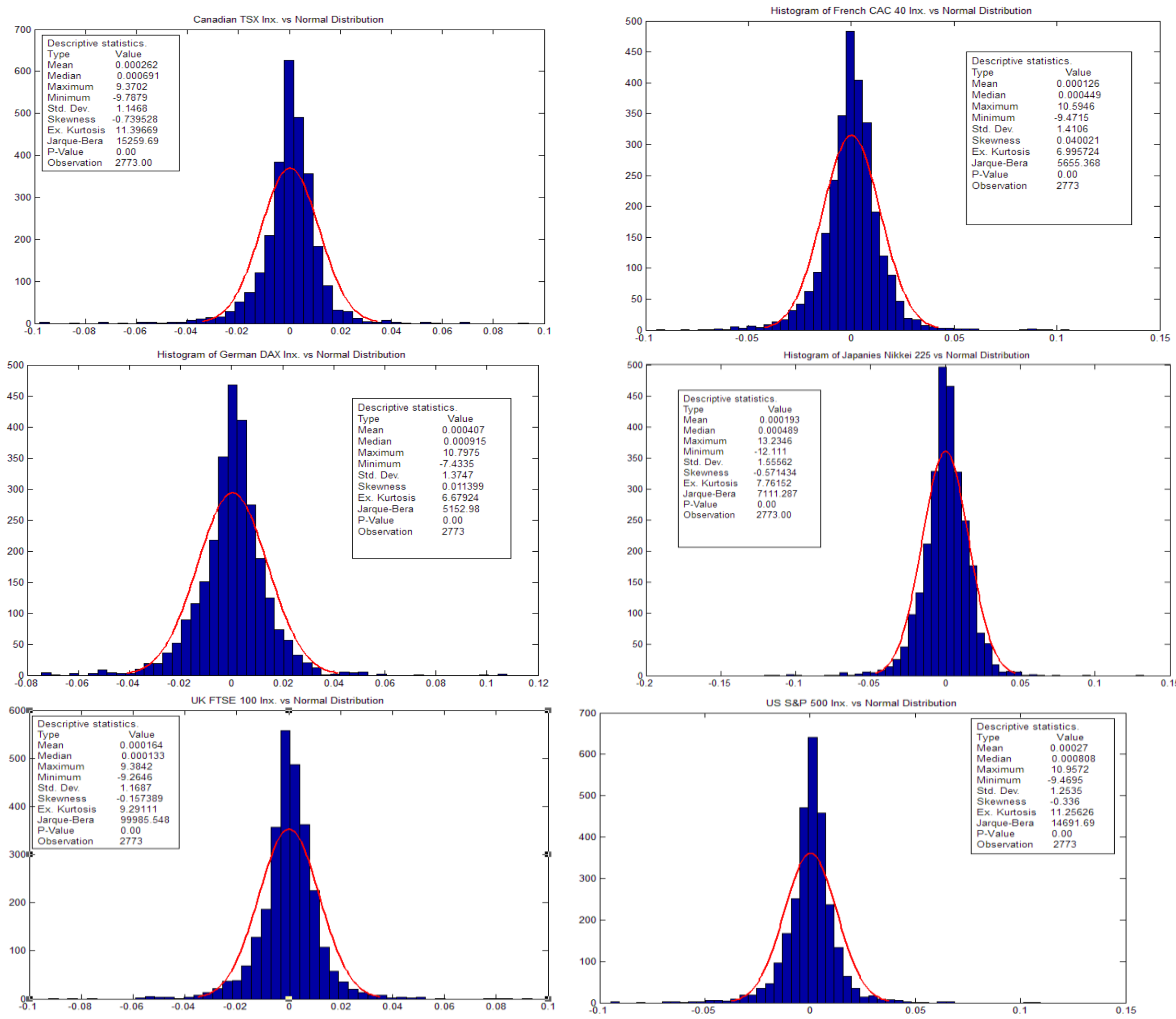

Figure. 2: Histograms and Normal distribution. The histograms and theoretical normal (red line) returns of stocks indexes. Sample run from June 3, 2003 to March 31, 2014. Visually we can claim that the distribution of all indices is similar to a *t-student* distribution. Additionally, descriptive statistics calculated for the whole period is reported in small boxes. The values of Kolmogorov-Smirnov test is also lower that 5% (is not reported).

Source: Own study.

Fig. 1 shows the histograms for each index with the theoretical Gaussian and *t*-Student probability density functions. These histograms seem symmetric. Therefore, in this paper, we consider only symmetric distributions. For all of the considered indices, the excess kurtosis statistics is very large, implying that the distribution of these returns has a much thicker tail than the normal distribution. Similarly, the Jarque–Bera statistics is also very large and statistically significant, disallowing the assumption of normality.

The outcome of daily various VaR methods based on a range of confidence levels for the TSX, CAC40, DAX, Nikkei, FTSE100, S&P500 are reported in table 1.

Table 1**:** Values of VaR estimated based on various techniques - from a low to a high level of confidence.

Quantile	Index	HS	GARCH	M-GARCH	E-GARCH	GJR-GARCH	A-GARCH	FHS	EVT
90%	TSX	1,13%‡	1,13%	1,13%	1,07%	0,98%	1,01%	1,12%	1,13%
	CAC	1,49%	1,48%	1,48%	1,40%	1,47%	1,49%	1,53%	1,49%
	DAX	1,50%	1,50%	1,50%	1,47%	1,37%	1,47%	1,50%	1,50%
	Nikkei	1,68%	1,69%	1,69%	1,76%	1,60%	1,67%	1,72%	1,69%
	FTSE	1,17%	1,16%	1,16%	1,04%	0,97%	1,09%	1,20%	1,17%
	S&P	1,21%	1,20%	1,20%	1,12%	0,98%	1,06%	1,24%	1,21%
95%	TSX	1,75%	1,76%	1,75%	1,59%	1,17%	1,60%	1,67%	1,75%
	CAC	2,23%	2,20%	2,18%	2,07%	2,11%	1,10%	2,25%	2,23%
	DAX	2,12%	2,11%	2,11%	2,12%	1,87%	1,12%	2,22%	2,12%
	Nikkei	2,37%	2,36%	2,36%	2,51%	2,27%	1,43%	2,42%	2,37%
	FTSE	1,78%	1,75%	1,75%	1,55%	1,19%	1,37%	1,78%	1,79%
	S&P	1,84%	1,85%	1,84%	1,58%	1,29%	1,23%	1,90%	1,85%
97.5%	TSX	2,20%	2,18%	2,18%	1,88%	1,50%	1,58%	2,09%	2,21%
	CAC	2,77%	2,77%	2,76%	2,51%	2,65%	2,52%	2,81%	2,77%
	DAX	2,54%	2,52%	2,52%	2,64%	2,39%	2,51%	2,78%	2,55%
	Nikkei	2,89%	2,89%	2,88%	3,17%	2,73%	2,80%	2,94%	2,90%
	FTSE	2,34%	2,26%	2,26%	1,92%	1,57%	2,20%	2,24%	2,34%
	S&P	2,40%	2,39%	2,39%	2,01%	1,70%	2,35%	2,43%	2,41%
99%	TSX	3,48%	3,42%	3,39%	2,86%	2,42%	2,73%	3,10%	3,49%
	CAC	4,06%	3,97%	3,94%	3,44%	4,01%	4,07%	4,16%	4,08%
	DAX	4,10%	3,94%	3,91%	4,03%	3,58%	3,30%	4,12%	4,12%
	Nikkei	4,33%	4,26%	4,24%	5,20%	4,22%	5,04%	4,11%	4,34%
	FTSE	3,24%	3,19%	3,19%	2,20%	2,17%	2,30%	3,32%	3,26%
	S&P	3,78%	3,69%	3,63%	2,50%	2,55%	2,49%	3,72%	3,86%
99.95%	TSX	9,69%	8,95%	8,76%	7,74%	17,76%	15,39%	9,43%	9,79%
	CAC	9,08%	8,66%	8,54%	10,76%	10,12%	11,04%	12,59%	9,47%
	DAX	7,41%	7,39%	7,38%	12,49%	7,75%	12,36%	12,52%	7,43%
	Nikkei	11,85%	10,80%	10,45%	27,25%	23,18%	24,08%	12,70%	12,11%
	FTSE	8,96%	8,29%	8,17%	6,22%	6,29%	6,12%	10,44%	9,26%
	S&P	9,44%	8,83%	8,70%	6,55%	6,76%	6,59%	12,92%	9,47%

‡ it shows that with 90% confidence interval VaR of TSX index would not exceed 1.13% in next day. In other words, the loss of TSX index will not exceed more than 1.13% of its value one-day horizon.

Source: own study.

In this section we would like to explain in details the way we calculated VaR of the two most sophisticated methods e.g. Filtered Historical Simulation with bootstrap and then Extreme Value Theory.

Filtered Historical Simulation with Bootstrap technique

No dividend adjustments are explicitly taken into account. Then with encoding equation (12) in MATLAB we model an asymmetric Exponential GARCH model to the conditional variance. Next step is to implement code segment calculate the autoregressive order (1) plus Exponential GARCH (1, 1) model. So, implementing this technique will enable us to extract the filtered residuals and conditional variance from each index return. Obtaining filtered the model innovation from the indices return series; standardize each innovation by the corresponding conditional standard deviation. These SIs represent the underlying unit-variance, zero-mean, $i.i.d.$ series. The $i.i.d.$ character is necessary for bootstrapping, and lets the sampling procedure to safely prevent the drawbacks of sampling from a population in which consecutive observations are serially dependent. To make the innovations standardized we shall apply equation (13).

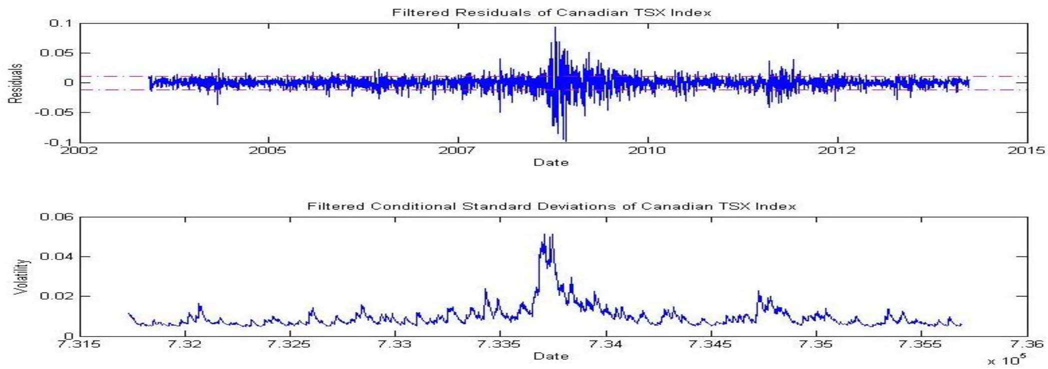

Figure 3: Plot of filtered residuals and volatility. The bottom plot exhibits existence of heteroskedasticity in the filtered residuals. The lower graph clearly illustrates the variations in volatility (heteroskedasticity) present in the filtered residual.

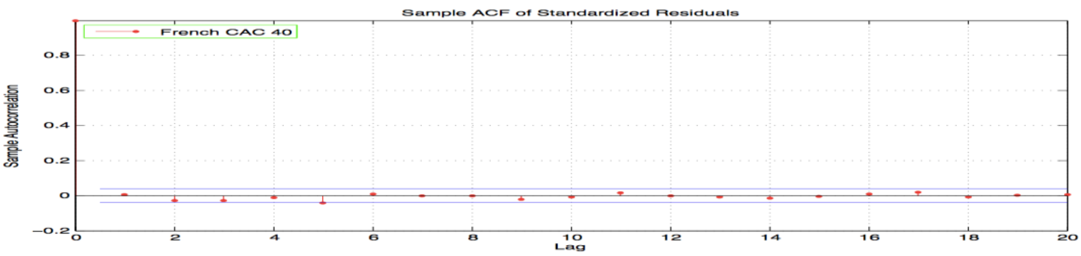

Figure 4: Sample ACF of French CAC index standardized residuals (similar results obtained for the rest of series).

As cited in section II, filtered historical simulation bootstraps SIs to create paths of future asset returns and, hence, makes no parametric assumptions about the probability distribution of those returns.

The bootstrapping procedures $i.i.d.$ SIs is in line with those obtained from the AR(1)-EGARCH(1,1) filtering process above. Exploiting the bootstrapped SIs as the identically and independently distribution input noise process, reestablish the autocorrelation and heteroskedasticity observed in the original index return series via the Econometrics Toolbox™ filter function. Obtaining simulated the returns of indices report the estimated value-at-risk at various confidence levels, over the one-day risk horizon is reported in Table 1 Part C.

For instance, based on filtered historical simulation method- Table 1 part C- figure 1.12% represent the value-at-risk of Canadian TSX index with 90 per cent confidence level, over one day horizon. In other words, it means that only with ten percent probability the VaR of Canadian TSX with exceed from 1.12% of its value over one-day horizon.

Extreme Value Theory

As mentioned in section II modeling the tails of a distribution with a generalized Pareto distribution necessitates the data under examination to be *approximately* $i.i.d$. To do so, we shall implement the same procedure similar to section II equations 13,14, and 15 in MATLAB to obtain our desirable data series. Results of Japanese Nikkei 225 index are summarized in figure 2. Results for rest of data e.g. Canadian TSX index, French CAC 40 index, German DAX index and US S&P 500 index can be found in appendix.

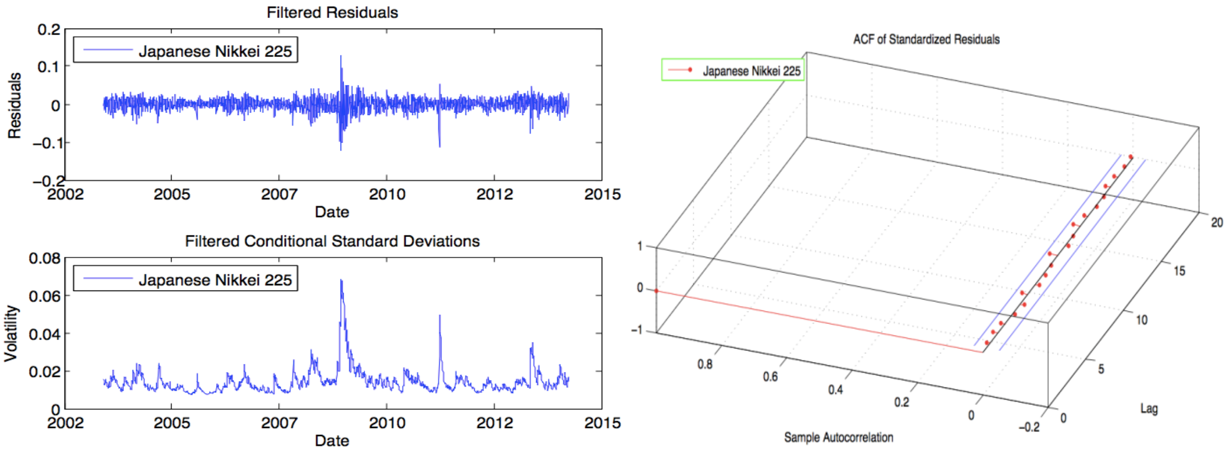

Figure 5: (Left) Filtered residuals and filtered conditional standard deviations of Japanese Nikkei 225 index. (Right) Shows the 3D ACF of standardized residuals of Japanese Nikkei index.

Having the standardized, identically and independently distribution of innovations from the previous stage, calculate the empirical cumulated distribution function of each index with a Gaussian kernel. This smoothes the cumulative distribution function estimates, removing the staircase shape of unsmoothed sample cumulated distribution functions. Although non-parametric kernel cumulated distribution function estimates are well fitted for the interior of the distribution where most of the data is concentrated, they tend to perform weakly when implemented to the upper and lower tails. Implement Extreme Value Theory to those innovations that fall in each tail to suitably calculate the each tail of the distribution. Precisely, find upper and lower u (threshold, the main function of) in implementation of equations 19,20,21, and 22 such that 10 per cent[‡‡‡] of the innovations in this paper are reserved for each tail. Afterward, based on mentioned method in section II, we shall apply Peak Over Threshold method as following.

Fit the amount by which those extreme innovations in each tail fall above the determined u to a parametric generalized Pareto distribution by maximum likelihood. Finally given the exceedances in each tail, optimize the negative log-likelihood function to estimate the shape

‡‡‡ The value of threshold is optional. Though, the sample mean excess function (MEF) is applied in some papers to determine the value of threshold more appropriately. Additionally, Neftci 2000 proposed another methods in his paper.

parameter (ξ) and sclae parameter (k) via MATLAB Econometrics Toolbox™ and then plug their values in equation (18) to estimate VaR based on EVT with difference confidence level over one-day horizon. The results of implementing previous steps on Japanese Nikkei 225 index are summarized as it is depicted in table 1, Part D, that summarizes the estimated VaR of indices considered based on EVT for different confidence level for one-day horizon. For instance, figure 4,12% refers to VaR of German DAX index with 99% confidence level. In other words, it states that with 99% confidence level VaR of DAX index will not exceed 4,12% of value of German DAX index over next trading day.

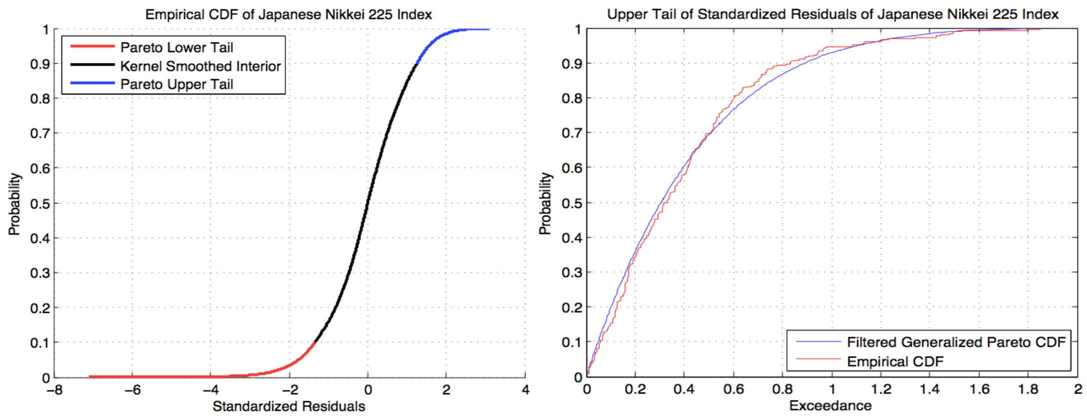

Figure 11: (Right)Empirical CDF of Japanese Nikkei 225 index. Plot of Pareto lower and upper tails. (Left) Shows Filtered Generalized Pareto CDF vs. empirical CDF of Japanese Nikkei index.

Source: own study

Backtesting results.

The VaR estimates, on April 01 2014, for all indices based on implementing methods considered, are presented in previous sections, whereas their test sample performance is evaluated in tables 2 and 3. This evaluation is based on one-step-ahead forecasts that have been produced from a series of rolling samples with a size 1500 observation and we shall base test on the 10%, 5%, 2.5%, 1% and 0.05% daily VaR for various methods and test have been suggested in section II.5 for evaluating VaR model accuracy. In this paper, we implement tests (equations 21 and 22) for covering probability.

In Table 5, we present the test statistics for the conditional test sample performance of various methods considered. The main evidence from this backtesting exercise is that the models perform equally well at low confidence level (e.g. from 90% up to 97.5%). However from the 99% level and beyond the superiority of the extreme values technique clearly emerges since it is the only method where not a single case exists with statistically significant forecasting failures. Looking at the all indices, except EVT, Historical Simulation and Filtered Historical Simulation also performed very satisfactory which were beyond expectation. Historical Simulated estimate VaR of four indices for all confidence intervals precisely, namely TSX Canadian Index, French CAC 40 index, German DAX and Japanese Nikkei 225 index. Filtered Historical Simulation, except one case, estimated VaR in all confidence level and for all indices precisely.

In this experiment among all GARCH model, only GARCH (1, 1) model and to some lower extend MGARCH perform accurate. The results for EGARCH and GJR-GARCH are really poor in our experiment for all indices.

Likelihood ratio tests statistics for the conditional LR_{cc}, Equation (26), out-of-sample performance of various methods in different confidence level of indices considered.

Model	p	90%	95%	97,50%	99%	99,95%
	test	LRcc	LRcc	LRcc	LRcc	LRcc
HS	TSX	0,25	0,02	1,09	3,53	0,42
	CAC 40	1,25	0,12	0,26	3,22	0,38
	DAX	0,74	0,92	2,71	0,17	0,38
	Nikkei 225	0,34	0,74	2,6	3,24	0,38
	FTSE 100	0,34	0,23	1,09	1,23	17,05
	S&P 500	0,73	0,01	1,04	5,23	2,36
GARCH (1,1)	TSX	0,04	0,53	1,08	4,82	1,8
	CAC 40	0,72	0,09	0,37	0,42	0,53
	DAX	0,12	0,04	0,34	0,01	2,83
	Nikkei 225	0,15	0,89	1,24	0,03	2,83
	FTSE 100	0,01	1,01	1,24	1,45	1,03
	S&P 500	2,34	1,34	1,24	2,87	0,02
DCC-M GARCH	TSX	0,17	0,76	7,75	0,83	0,62
	CAC 40	0,04	5,87	1,71	0,28	0,62
	DAX	0,34	0,24	0,71	0,92	0,34
	Nikkei 225	0,06	0,23	3,24	1,05	0,71
	FTSE 100	1,37	0,23	3,22	0,33	3,34
	S&P 500	0,06	2,2	0,15	0,02	1,05
EGARCH(1,1)	TSX	0,74	2,5	0,02	4,53	9,33
	CAC 40	0,01	0,01	0,02	0,48	32,25
	DAX	0,08	3,33	2,94	0,42	13,28
	Nikkei 225	0,22	0,02	1,64	0,01	21,03
	FTSE 100	0,24	0,02	1,85	9,52	14,42
	S&P 500	2,9	0,02	20,04	7,97	17,93
GJR-GARCH (1,1)	TSX	0,19	1,08	2,95	0,37	23,84
	CAC 40	1,53	1,08	1,98	0,51	18,93
	DAX	7,39	2,34	1,68	0,51	18,03
	Nikkei 225	0,33	0,67	1,03	4,67	12,04
	FTSE 100	0,19	0,98	5,48	1,53	25,25
	S&P 500	0,18	0,67	11,03	9,33	21,93
FHS	TSX	2,01	1,45	1,13	0,01	9,33
	CAC 40	0,47	0,02	0,13	0,67	1,38
	DAX	0,08	3,65	0,72	0,31	0,02
	Nikkei 225	0,01	0,01	0,02	0,22	1,36
	FTSE 100	0,67	0,01	1,84	1,33	0,83
	S&P 500	0,35	0,23	0,17	1,98	1,25
EVT	TSX	0,89	0,89	0,17	1,22	0,28
	CAC 40	0,34	1,08	0,17	1,22	0,23
	DAX	0,15	1,09	1,44	0,03	0,07
	Nikkei 225	0,54	2,67	0,73	1,02	0,05
	FTSE 100	1,26	1,07	1,95	1,62	0,05
	S&P 500	0,73	0,17	2,03	0,62	0,23

Table 2: Backtesting period: January 04, 2009 to March 31, 2014. Red numbers indicate significance at the 95% level. LR_{cc} is χ^2 with 2 DoF.

Source: own study.

Number of exceedances, F, and 95% LR_{uc} non-rejection confidence regions for indices considered.

Index	Quantile	Failure (LRuc)	HS	GARCH(1,1)	MGARCH	EGARCH	GJR-GARCH	AGARCH	FHS	EVT
TSX	90%	11<F<29	22	20	21	27	27	24	19	19
CAC			17	18	18	23	27	19	20	19
DAX			21	20	16	19	18	22	22	19
Nikkei			16	18	16	21	24	20	15	17
FTSE			19	21	24	22	19	21	20	20
S&P			18	17	23	19	26	26	21	23
TSX	95%	7<F<23	14	12	17	19	19	14	13	16
CAC			9	11	15	15	17	19	13	14
DAX			11	12	13	16	16	11	14	13
Nikkei			13	12	11	19	17	18	14	14
FTSE			10	13	12	17	19	18	15	16
S&P			14	12	11	15	15	16	14	16
TSX	0,9750	3<F<12	7	9	10	12	13	10	8	7
CAC			5	10	9	9	9	9	7	9
DAX			6	9	8	8	10	12	10	10
Nikkei			6	9	11	12	10	9	8	9
FTSE			8	11	10	12	12	7	9	9
S&P			7	8	8	11	12	11	8	10
TSX	0,9900	F<8	2	8	2	8	9	8	2	1
CAC			3	2	2	8	5	8	1	1
DAX			2	2	1	2	8	5	2	0
Nikkei			2	1	2	9	2	3	2	0
FTSE			1	2	1	8	8	6	1	1
S&P			2	2	1	9	8	9	2	1
TSX	0,9950	F<2	0	2	2	3	2	3	1	0
CAC			1	2	2	2	2	2	1	0
DAX			0	1	0	2	2	2	2	0
Nikkei			0	2	1	2	2	3	1	0
FTSE			2	2	2	3	3	2	1	0
S&P			1	1	0	3	2	2	2	0

Table 3: Backtesting sample period: April 01, 2009 to March 31 2014. Red figures indicate statistically significant underestimation or overestimation of value-at-risk. F is the number of failures that could be observed without rejecting the null that the models are correctly calibrated at the 95% level of confidence.

In Table 3, we present the number of exceedances in each case and compare them with an interval of numbers that would be consistent with the probability level under which the VaR estimates have been produced. Those intervals for LR_{uc}test have been derived from equation (20). Again, we reconfirm for all indices the previous results whereas at high confidence levels the EVT method are generating the best performance. The Parametric models have also recorded a similar failure whilst, Historical Simulation and Filtered Historical Simulation recorded a much better results.

Summary of results and conclusion

Value-at-Risk (VaR) is one of the most popular risk measures used in realm of finance. The precise estimation of VaR is a crucial task for any financial institution, in order to arrive at the accurate capital requirement in response to framework of Basel II and meet the adverse behaviour of the market. We have illustrated the implementation of Historical Simulation, GARCH, EGARCH, AGARCH, GJR-GARCHEVT, DCC-MGARCH, Filtered Historical Simulation and finally Extreme Value Theory that are a combination of traditional and new tools toward risk measurement in a univariate distribution framework. There are different attitudes toward estimating Value-at-Risk, and most of them falsely assume that stock returns come from a normal distribution or multivariate normal distribution in the case stock portfolio. The three approaches that we illustrated in this paper are (a) Parametric approach that uses a long series of stock return, giving the same weight to each of them, assumes that the empirical distributions observed in the past mirrors future changes, (b) Non-parametric approach in which assume some assumption associated with behavior of stock returns. For instance, in this approach they assign the parameter of β to market risk as well as parameter λ to leverage effect and (c) Semi-parametric method that uses the non-parametric empirical distribution to capture the small risks and the parametric method based on EVT to capture large risks in result of rare events.

The use of EVT in the model improves the calculating of value-at-risk for extreme quantile because apart from modeling the fat tails it permits for extrapolation in the tails above the data series.

Our major conclusion is that the EVT outperform other techniques considered in this paper.

However Extreme Value Theory suffers from strong statistical underpinning and requiring a high level of programing and modeling skills either in MATLAB or R, meanwhile results are completely satisfactory and consistently reliable in different business cycles especially for high volatility periods. In our experience for a moderately calm period ,Apr. 2014, we estimate VaR of S&P 500 equal to 9.47 per cent with 99.95 per cent confidence level which seems reasonable for movements of these days stock indices and is in line with results of Berger (2013),Brooks el al. (2005), Neftci (2000),Raggad (2009),Lin et al., (2009), Bali (2007), Stelios et al. (2005), Abad el al., (2012), among others. And in line with Raggad (2009) Filtered historical Simulation and Historical Simulation perform satisfactory especially from low level of confidence, 90% to rather high level of confidence 99%.

All in all in our experiment, GARCH models did not exhibits a good performance in estimating VaR. meanwhile, GARCH (1, 1) and MGARCH exhibits a better performance especially in lower confidence level. My intuition about the reason for poor performance of GARCH can be probed in our data. It is fact that GARCH model are persistent to unordered movements in stock returns. Inclusion of a high volatile period like wake of 2008 financial crisis in our data negatively affects on predictability power of almost all GARCH models. Since in section II we detected property of leptokurtosis and negative skewness among the data and when the function form of parametric distribution has leptokurtosis and negative skewness, the empirical value-at-risk estimated at high confidence level (97.5, 99, 99.95) was greater than the VaR estimated by non-parametric and Semi-parametric methods. However, the opposite is yet correct at the lower confidence levels (0.90 and 0.95) in our experiment.

In this step, we find it suitable to suggest future research based on GARCH model if they want to estimates VaR for short-period it is better to take a shorter horizon time maximum 5 years to data under examination be a good representative of current market status. Because it is hardly possible that equity markets will return to their previous levels of volatility such as 2008 credit crisis within a short risk horizon like one or even next 10 trading days.

We suggest, for further research, to probe the performance of GARCH models in two homogeneous periods. A calm and a volatile period and compare their result of performance of GARCH model in each period.

Furthermore, we suggests that further work needs to be done to test the sensitivity of EVT model based on the choice of threshold level, u,.

References:

Abad A., Benito S., López L. (2013). A comprehensive review of Value at Risk methodologies, Journal of the Spanish Finanical Econometrics.

Andersen T. G., Bollerslev T. (1998). Answering the Skeptics: Yes, Standard Volatility Models do Provide Accurate Forecasts, International Economic Review, Vol. 39, No. 4.

Bali R. (2003). Seasonality in ex dividend day returns, Applied Economics Letters, Vol. 10, No. 14.

Barone-Adesi G., Giannopoulos K., Vosper L. (1999). VaR without correlations for nonlinear portfolios, Journal of Futures Markets 19, pp. 583–602.

Black, F. (1976). Studies in stock price volatility changes. In: Proceedings of the 1976 Business Meeting of the Business and Economics Statistics Section, American Association, pp. 177–181.

Beirlant J., Teugels J.L., Vyncker P. (1996). Practical Analysis of Extreme Values. Leuven University Press, Leuven, Belgium.

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return, Review of Economics and Statistics 69, pp. 542–547.

Bollerslev T. (1986). Generalized Autoregressive Conditional Heteroskedasticity, Journal of Econometrics 31, 307-327, North-Holland.

Boudoukh J., Richardson M., Whitelaw R. (1998). The best of both worlds, Risk 11(May), pp. 64-67.

Brooks C., Clare A.D., Dalle Molle J.W., Persand G. (2005). A comparison of extreme value theory approaches for determining value at risk, Journal of Empirical Finance 12, 339– 352.

Butler, J.S., Schachter, B. (1998). Estimating value at risk with a precision measure by combining kernel estimation with historical simulation, Review of Derivatives research 1, pp. 371-390.

Dave R., Stahl G. (1997). On the accuracy of VaR estimates based on the varianvce-covariance approach, mimeo, Olsen & Associates.

Davison A. C. (1990). Models for Exceedances over high threshholds, Journal of the Royal Statistics Society, Vol. 52, No. 3, pp. 393-442.

Ding Z., Granger C.W.J, Engle R. F. (1993). A long memory property of stock market returns and a new model, Jouranal of Empirical Finance 1, pp. 83-106, North-Holland.

Engle, R., Manganelli, S. (2004). CAViaR: conditional autoregressive value at risk by regression quantiles. Journal of Business & Economic Statistics 22, 367–381.

Engle R. (2002). Dynamic conditional correlation – A simple class of Multivariate GARCH models, Journal of Business and Economic Statistics.

Engle R. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, Econometrica, Vol. 50, No. 4., pp. 987-1007.

Fernandez V. (2005). The International CAPM and a wavelet-based decomposition of Value at Risk, Documentos de Trabajo 203, Centro de Economía Aplicada, Universidad de Chile.

Gencay, Ramazan, Selcuk, Faruk. (2006). "Overnight borrowing, interest rates and extreme value theory", European Economic Review, Elsevier, vol. 50(3), pages 547-563, April.

Gencay, Ramazan, Selcuk, Faruk. (2004). Extreme value theory and Value-at-Risk: Relative performance in emerging markets, International Journal of Forecasting, Elsevier, vol. 20(2), pages 287-303.

Glosten L. R., Jagannathan R., Runkle D. E. (1993).On the Relationship between the Ecpected Value and the Volatility of the nominal Excess Return on Stocks, The journal of Finance, Vol. 48, no. 5, pp. 1779-1801.

Hull, J., White, A. (1998). Incorporating volatility updating into the historical simulation method for value-at-risk, Journal of Risk 1, pp. 5–19.

Jorion, P. (2001). Value at Risk: The New Benchmark for Managing Financial Risk. McGraw-Hill.

Jorion, P. (1997). Value at Risk: The New Benchmark for Controlling Market Risk. Irwin, Chicago, IL.

Jorion, P. (1990). The exchange rate exposure of U.S. multinationals. Journal of Business 63, pp. 331–345.

J.P.Morgan, RiskMetrics - Technical documents.

McNeil A. (1998). Calculating Quantile Risk Measures for Financial Time Series Using Extreme Value Theory. Department of Mathematics, ETS, Swiss Federal Technical University E-Collection, http://e-collection.ethbib.etchz.ch/

Marohn, F. (2005). Tail index estimation in models of generalized order statistics, Communications in Statistics: Theory & Methods, 34:5, pp. 1057-1064.

Merton R. C., 1980, ON ESTMATING THE EXPECTED RETURN ON THE MARKET, Journal of Financial Economics 8, pp. 323-361, North-Holland Publishing Company.

Neftci, S.N. (2000). Value at Risk calculations, extreme events, and tail estimation. The Journal of Derivatives 7, 23–37.
Nelson, Daniel B. (2005). Conditional heteroskedasticity in asset returns: a new approach, Econometrica, vol. 59, No.2, 347-370.
Pagan, A., Schwert, G. (1990). Alternative models for conditional stock volatility. Journal of Econometrics 45, pp. 267–290.
Perignon, C., Smith, D. (2010). The level and quality of Value-at-Risk disclosure by commercial banks, Journal of Banking and Finance 34, pp. 362-377.
Taylor S.J., Xu X. (1997). The incremental volatility information in one million foreign exchange quotations, Journal of Empirical Finance 4, pp. 317-340.
Zakoian J. M. (1994). Threshold heteroskedastic models, journal of European Dynamics and Control, Vol. 18, Issue 5, pp. 931-955.

Appendix

Table 1: Lagged daily return

s regression of French CAC 40 Index.

	Coefficients	*SE*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	0.503	0.614	0.000	0.000
Lagged return	0.052	0.018	2.753	0.005	0.089	0.015

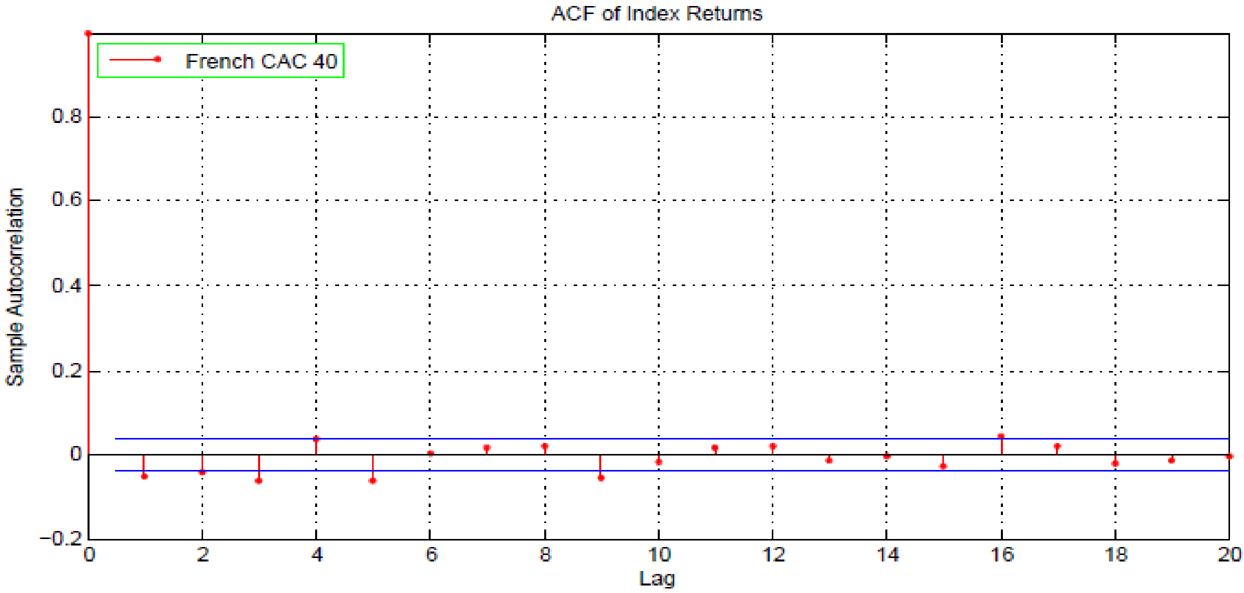

Figure 1: ACF of CAC 40 Index. It reveals that just first lag crosses the 95% bounds.

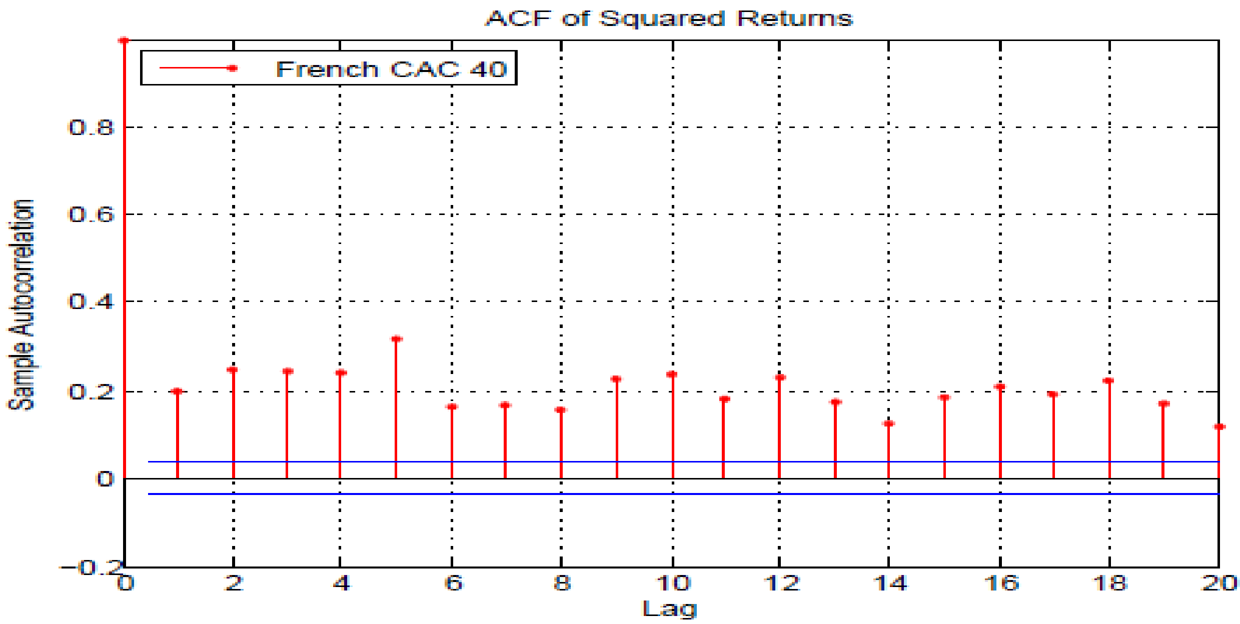

Figure 2: The sample ACF of the squared returns illustrates the degree of persistence in variance.

Table 2: Lagged daily returns regression of German DAX index

	Coefficients	*SE*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	1.570	0.116	0.000	0.000
Lagged return	0.006	0.019	0.341	0.732	0.043	0.030

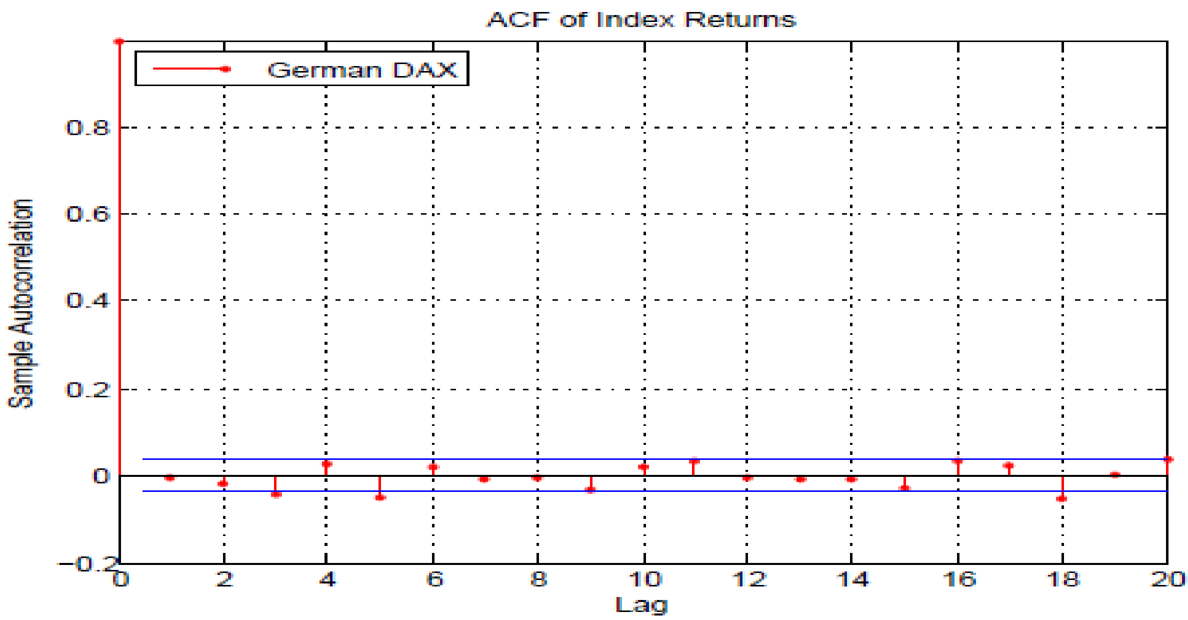

Figure 3: ACF of German DAX index shows breaking 95% bounds in several times.

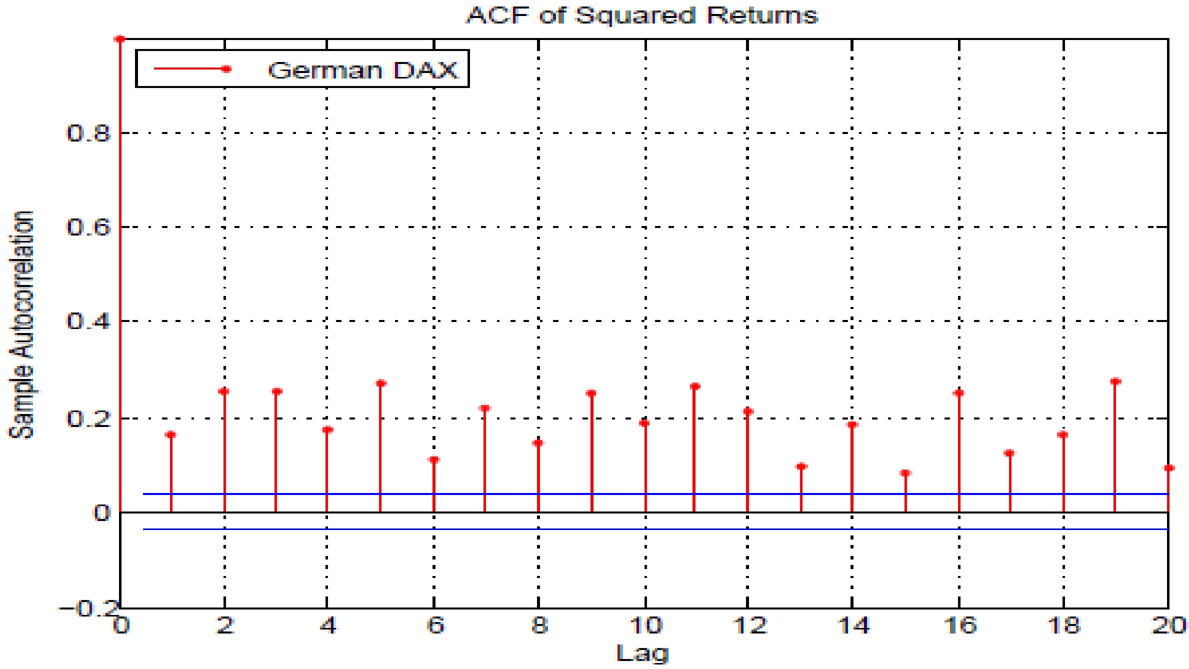

Figure 4: The sample ACF of the squared returns illustrates the degree of persistence in variance.

Table 3: Lagged daily returns regression of Japanese Nikkei 225 index.

	Coefficients	*SE*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	0.676	0.498	0.000	0.000
Lagged return	0.035	0.018	1.85	0.051	0.072	0.001

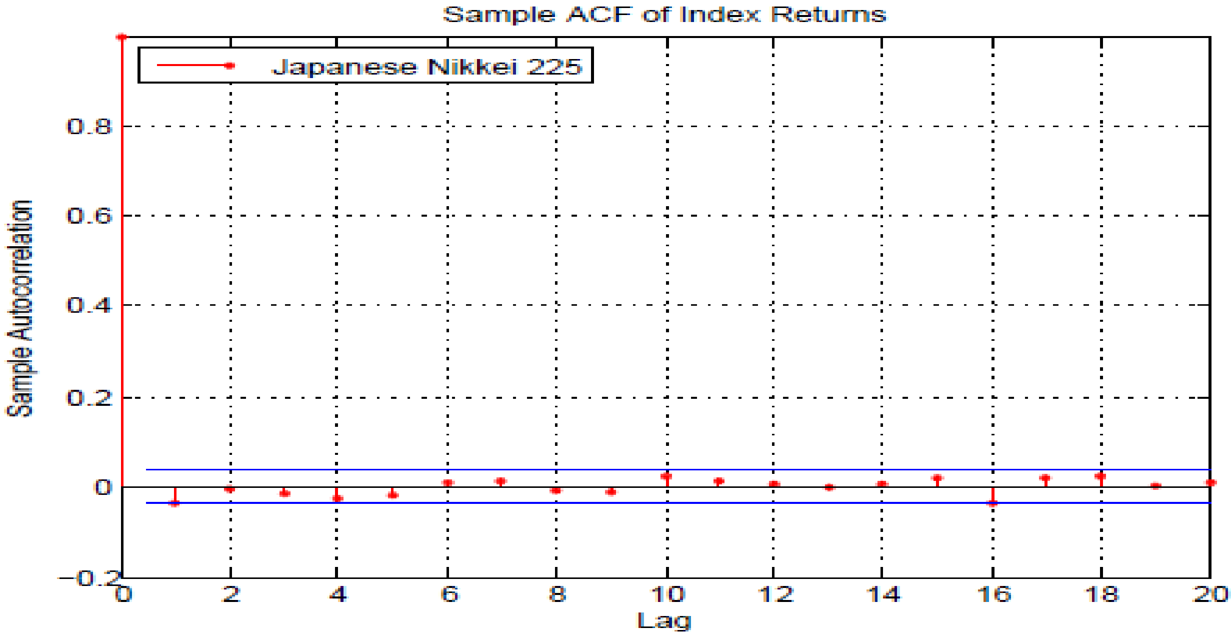

Figure 5: ACF of Japanese index really closing to beating 95% bounds just in first lag with p-value close to 5%.

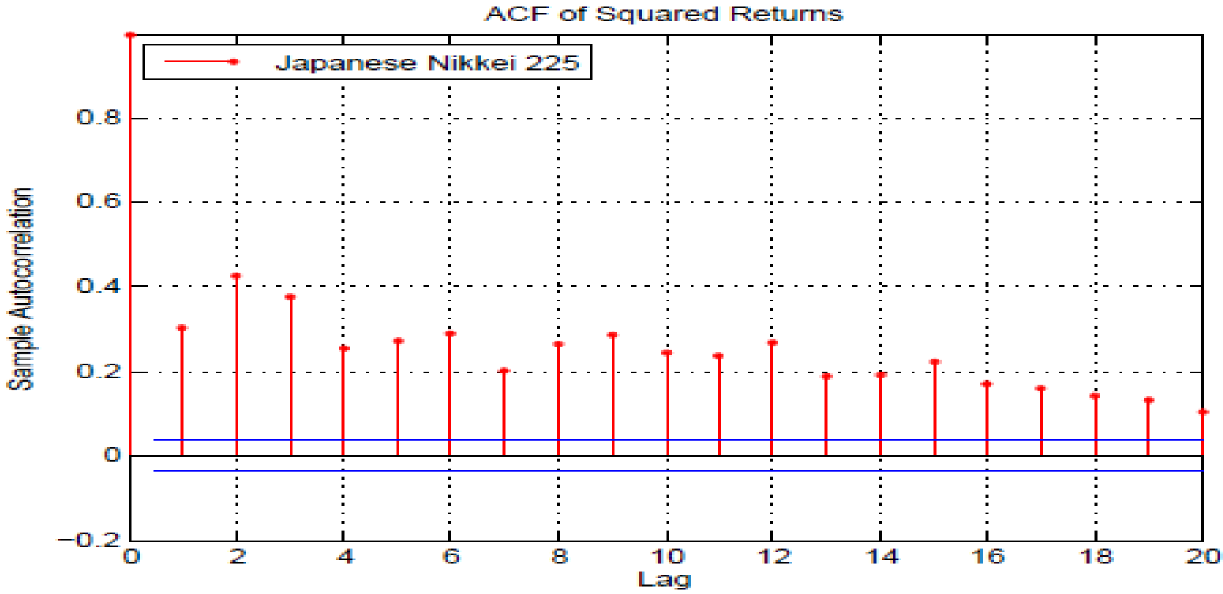

Figure 6: The sample ACF of the squared returns illustrates the high degree of persistence in variance until lag 12 and mild onward.

Table 4: Lagged daily returns regression of UK FTSE 100 index.

	Coefficients	*SE*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	0.765	0.444	0.000	0.000
Lagged return	0.048	0.018	2.560	0.010	0.085	0.011

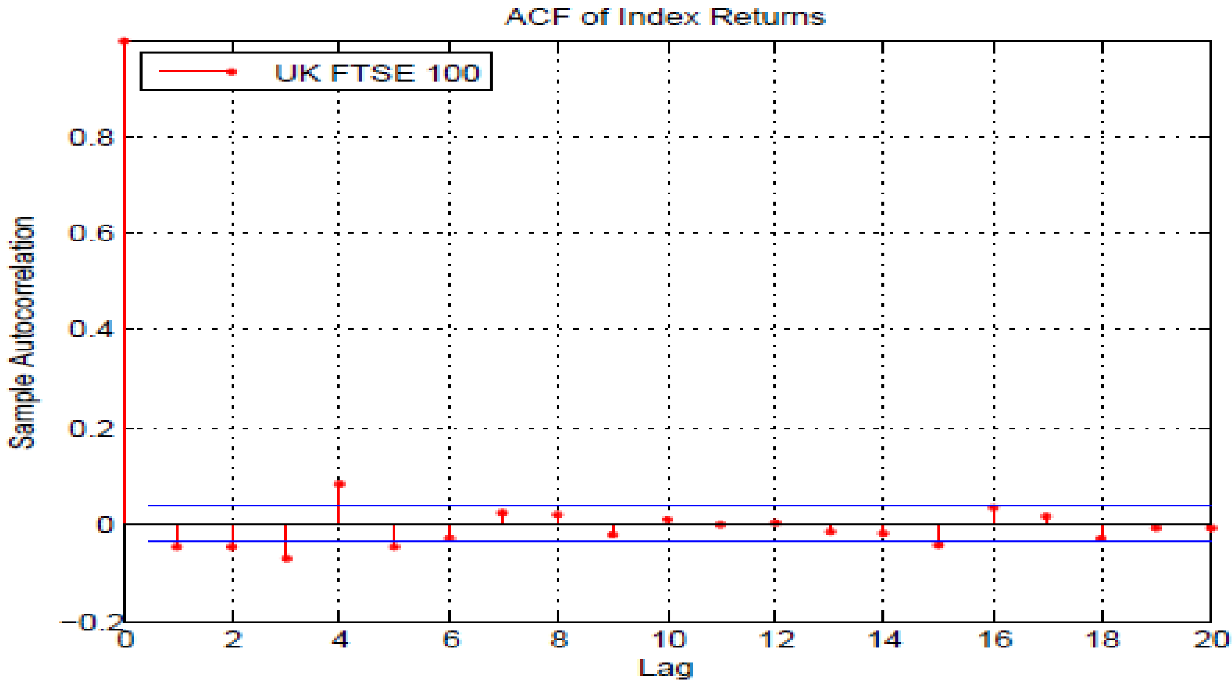

Figure 7: ACF of UK index exceeding 95% bounds just in first lag with a significant p-value equal to 1%.

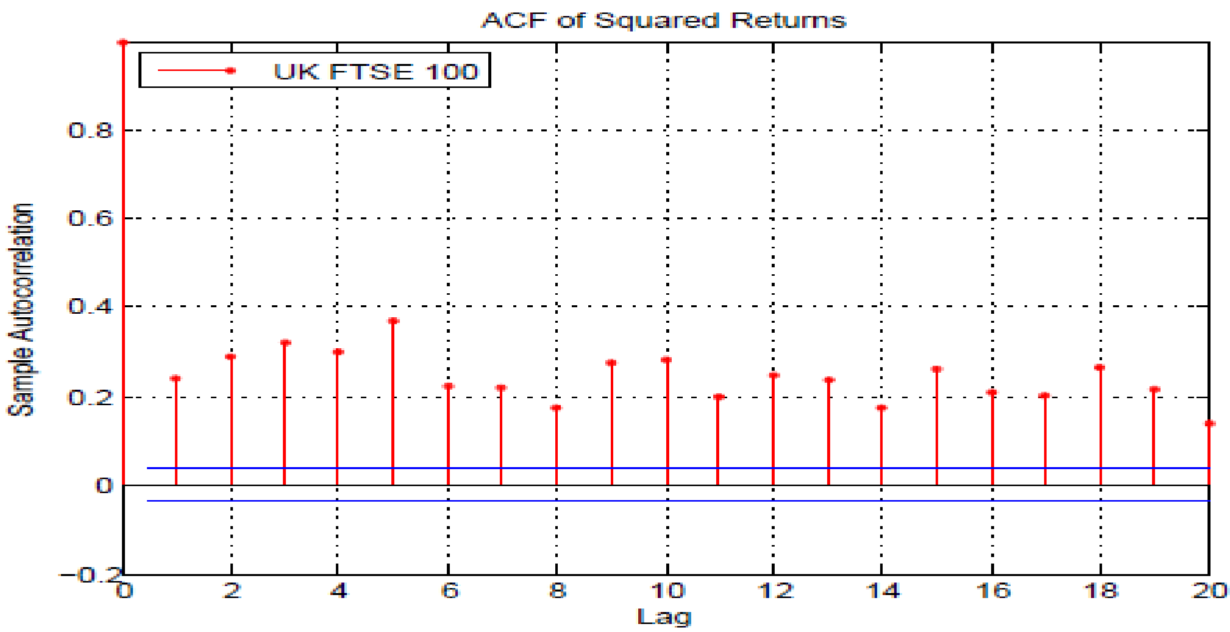

Figure 8: The sample ACF of the squared returns illustrates a high degree of persistence in variance.

Table 5: Lagged daily returns regression of US S&P 500 index.

	Coefficients	*SE*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.000	0.000	1.261	0.207	0.000	0.000
Lagged return	0.111	0.018	5.895	4.19E	0.148	0.074

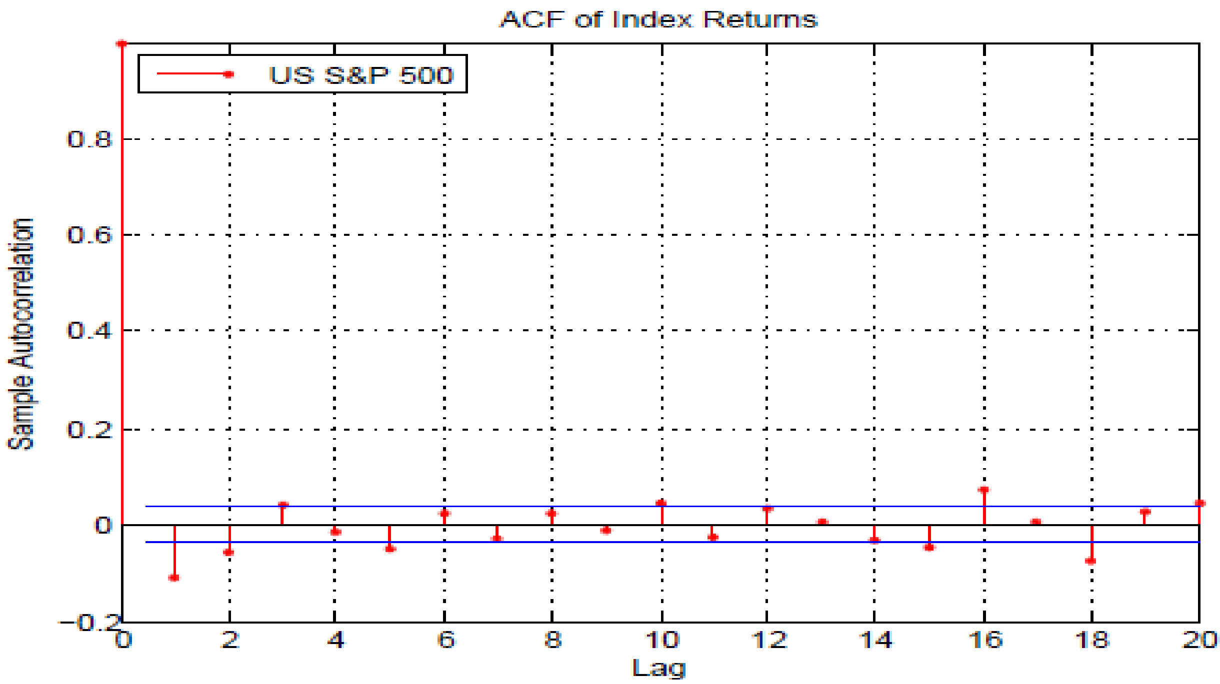

Figure 9: ACF of US S&P 500 Index. It reveals that just first lag crosses the 95% bounds.

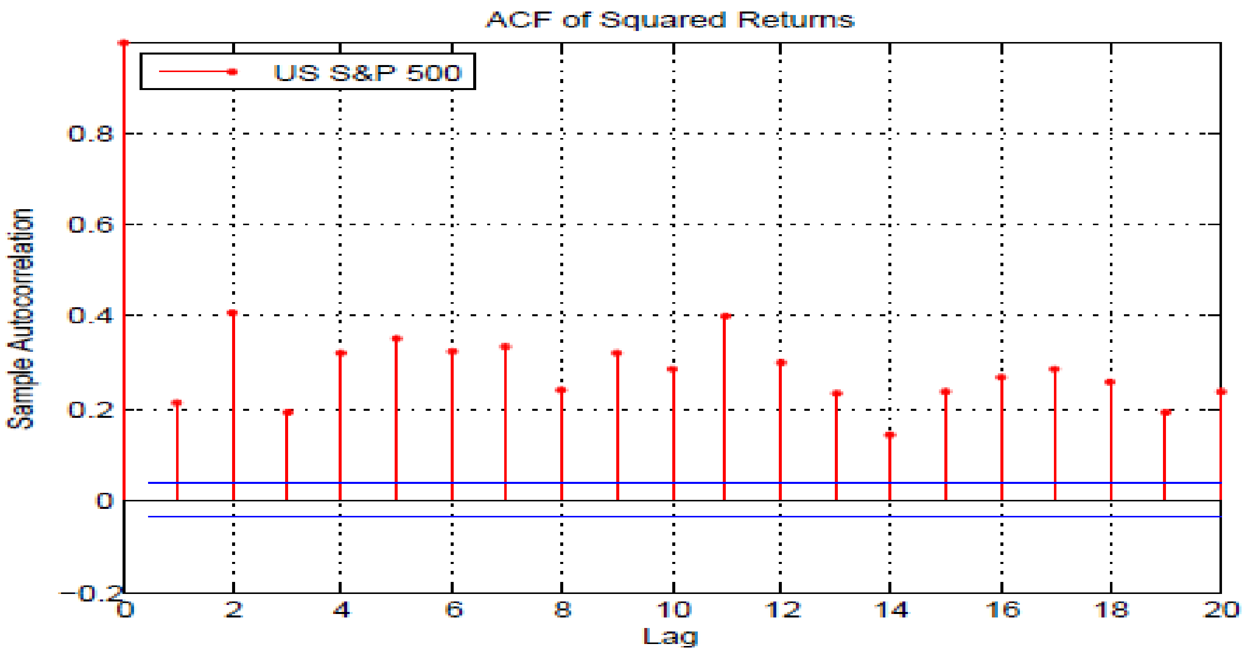

Figure 10: The sample ACF of the squared returns illustrates a high degree of persistence in variance.

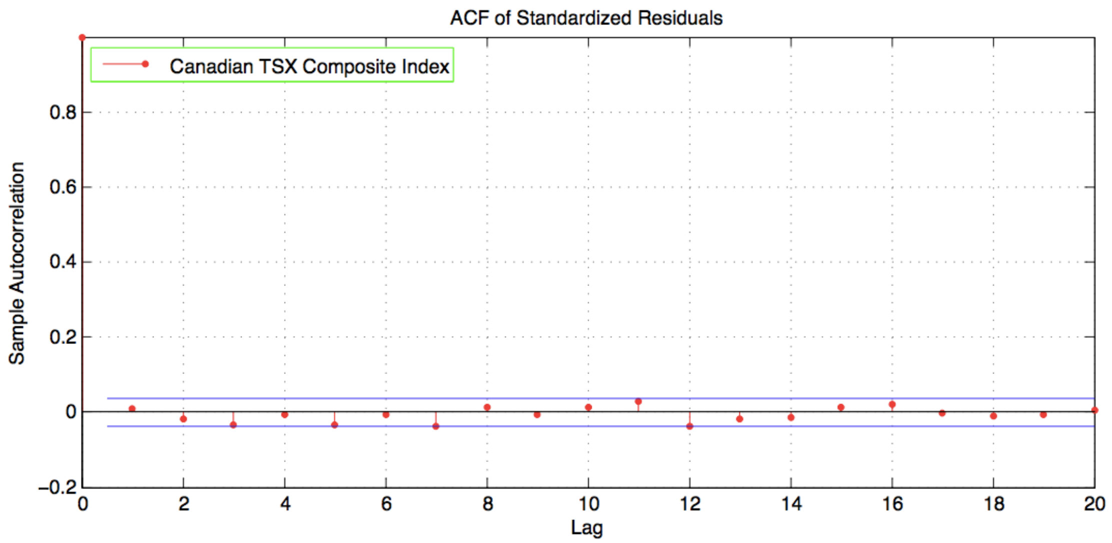

Figure 11: ACF of standardized residuals of Canadian TSX index. Source: Yahoo! Finance.

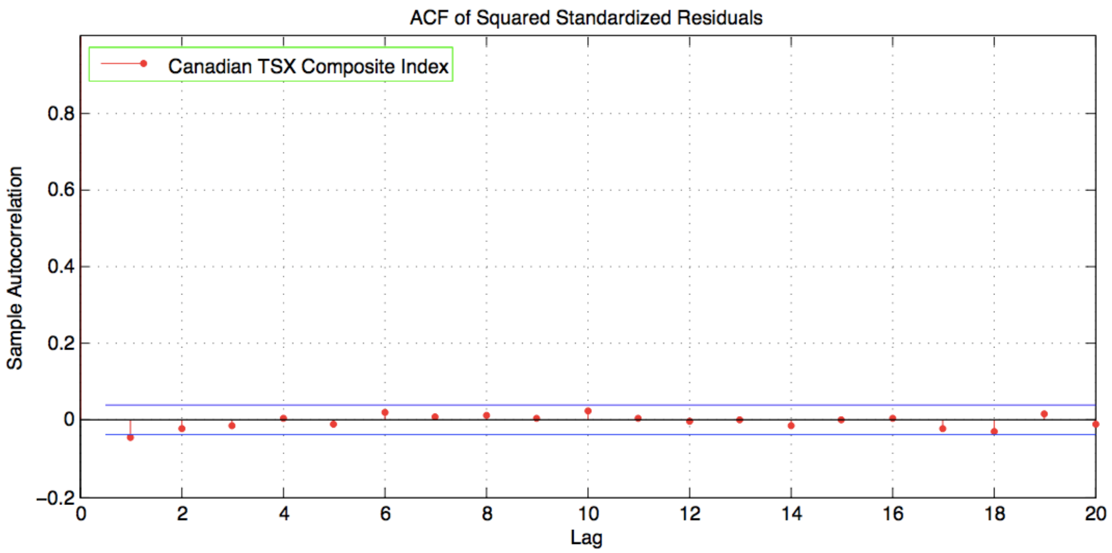

Figure 12: ACF of squared standardized innovations of Canadian TSX index. Source: Yahoo! Finance.

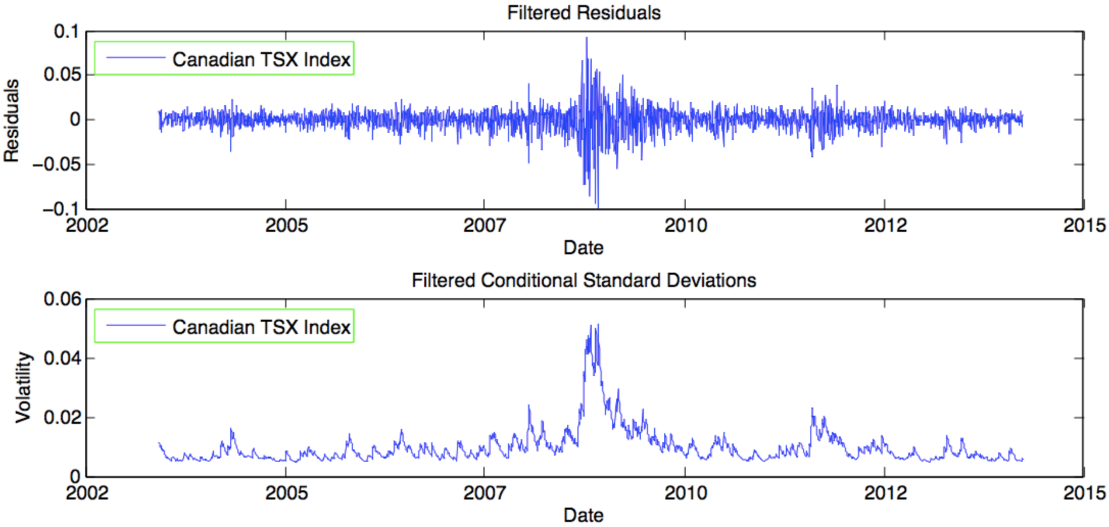

Figure 13: Plot of filtered innovations and filtered conditional standard deviations of Canadian TSX index. Source: Yahoo! Finance.

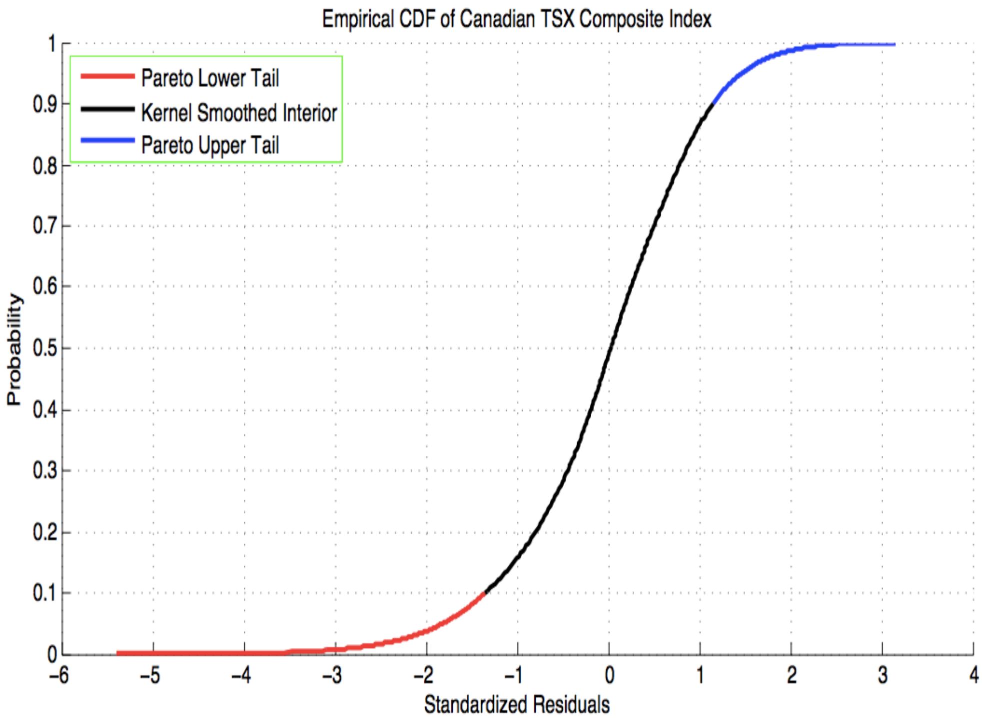

Figure 14: Empirical CDF of TSX index. Source: Yahoo! Finance.

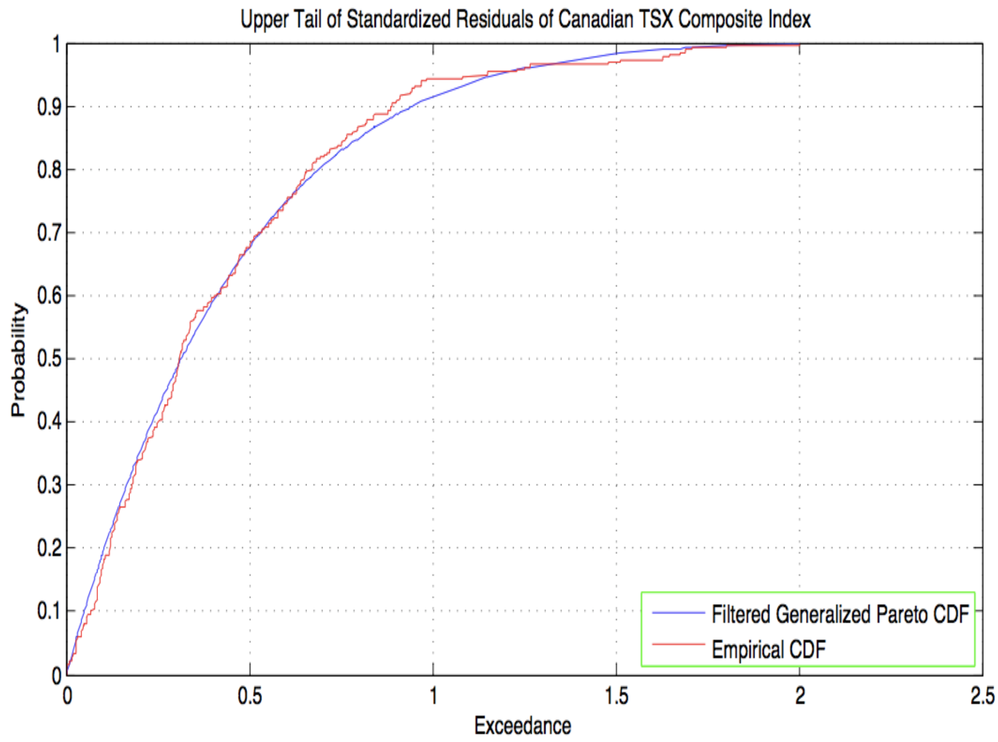

Figure 15: Filtered Generalized Pareto CDF v empirical CDF. Source: Yahoo! Finance.

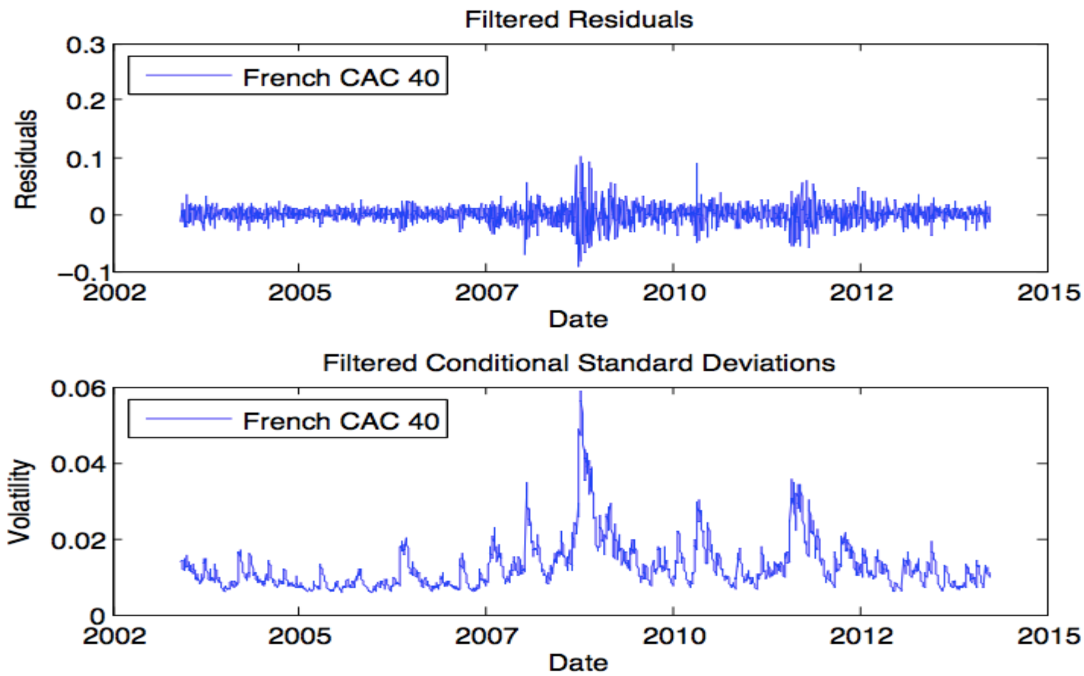

Figure 16: Filtered innovations and filtered conditional standard deviations of French CAC 40 index. Source: Yahoo! Finance.

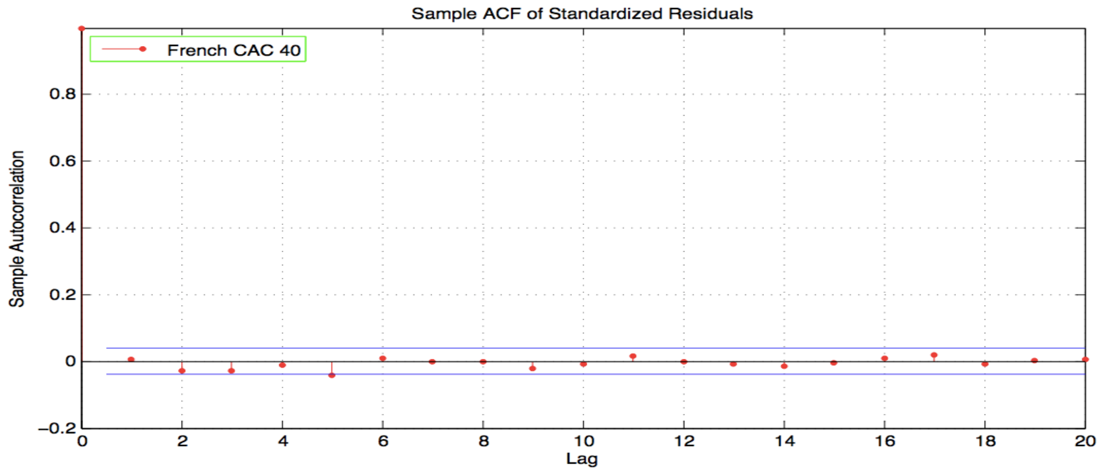

Figure 17: ACF of standardized innovations of French CAC 40 index. Source: Yahoo! Finance.

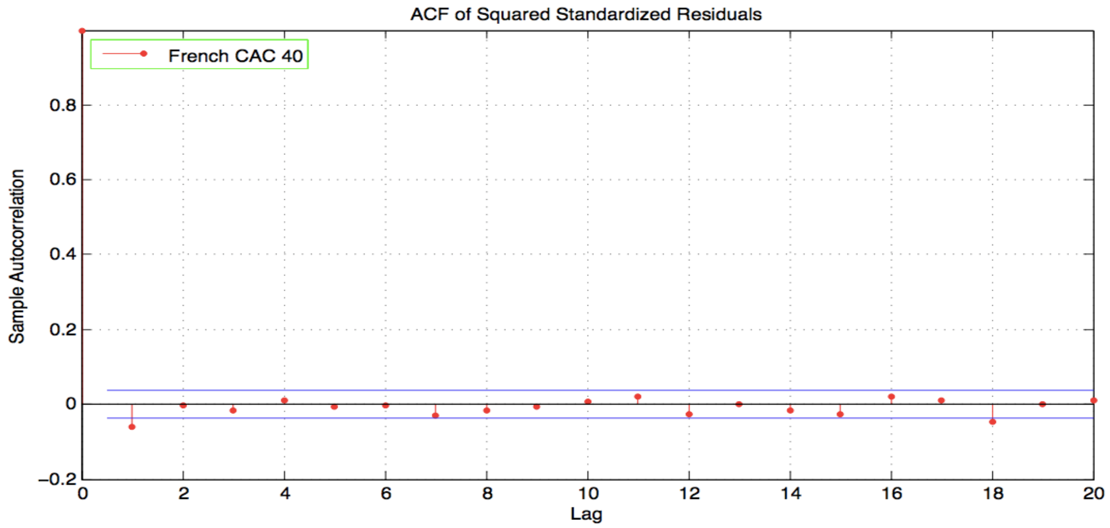

Figure 18: ACF of squared standardized residuals of French CAC 40 index. Source: Yahoo! Finance.

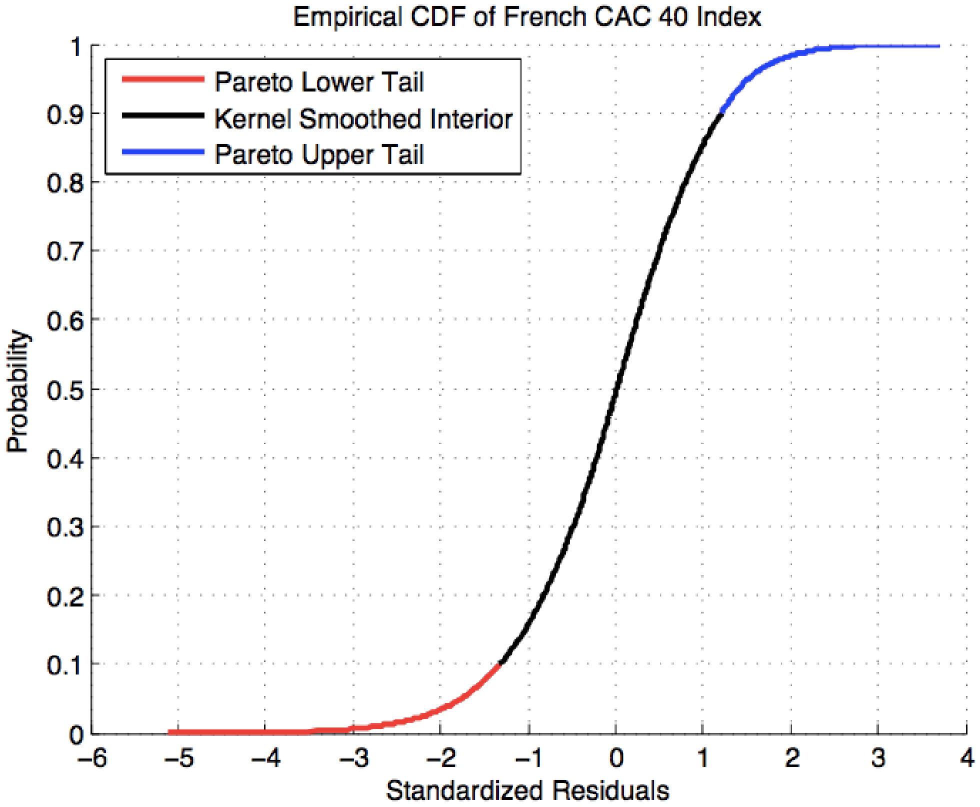

Figure 19: Empirical CDF of French CAC 40 index. Source: Yahoo! Finance.

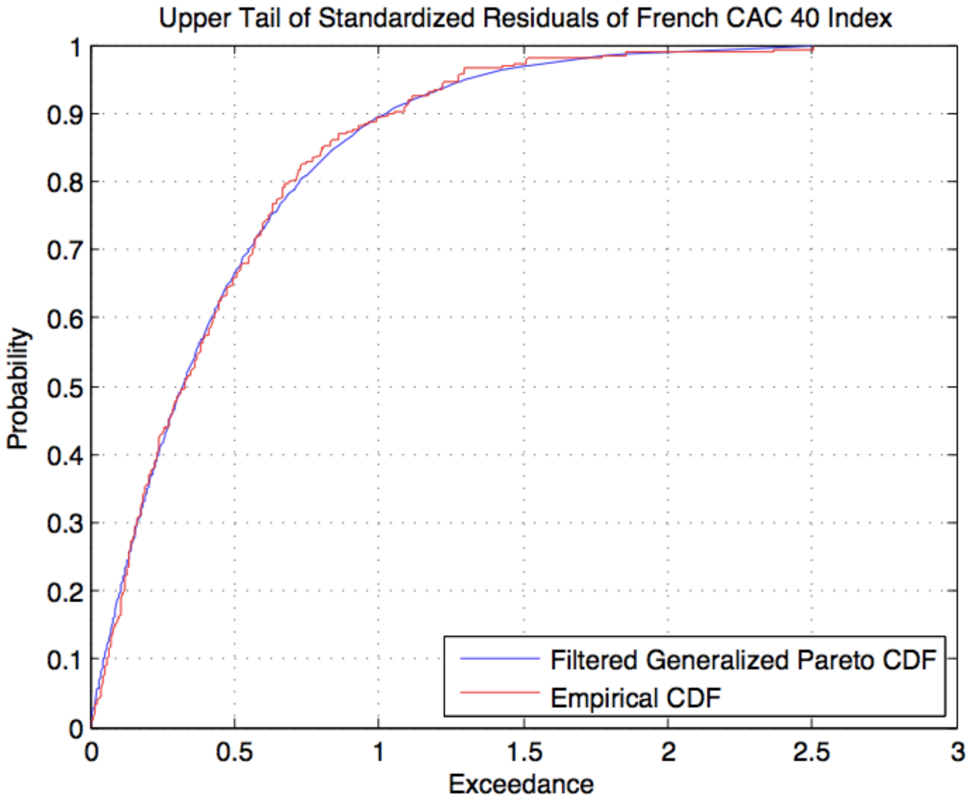

Figure 20: Filtered Generalized Pareto CDF vs empirical CDF of CAC index. Source: Yahoo! Finance.

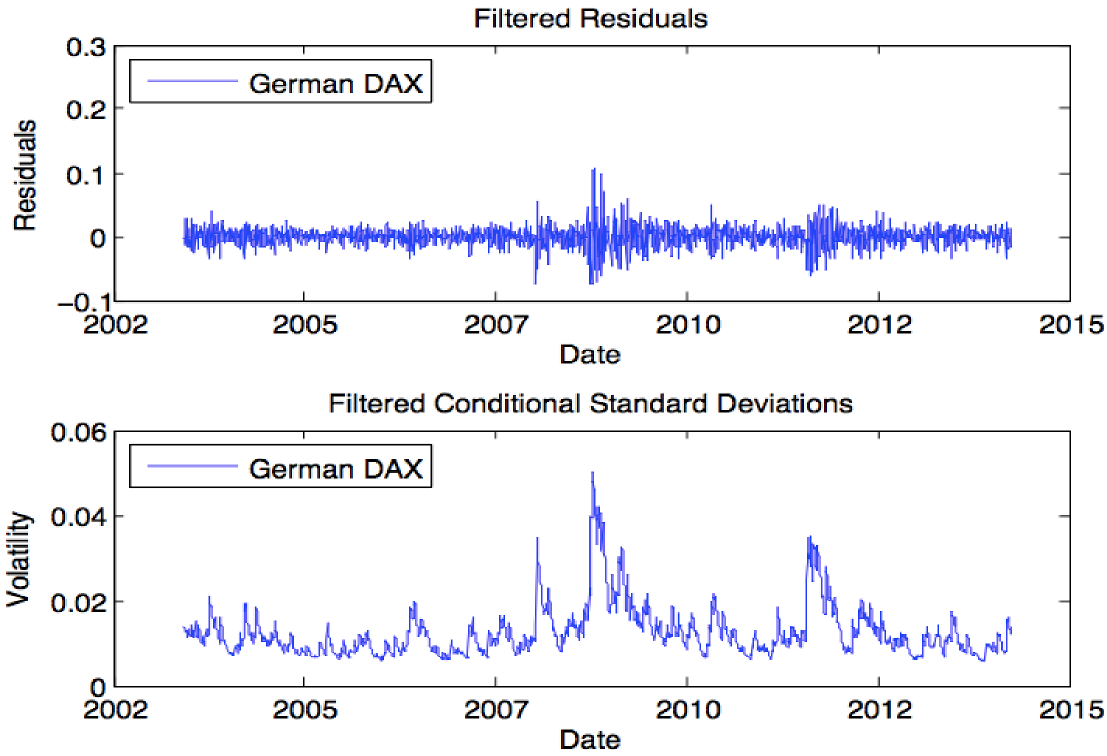

Figure 21: Plot of filtered innovations and filtered conditional standard deviation of German DAX index. Source: Yahoo! Finance.

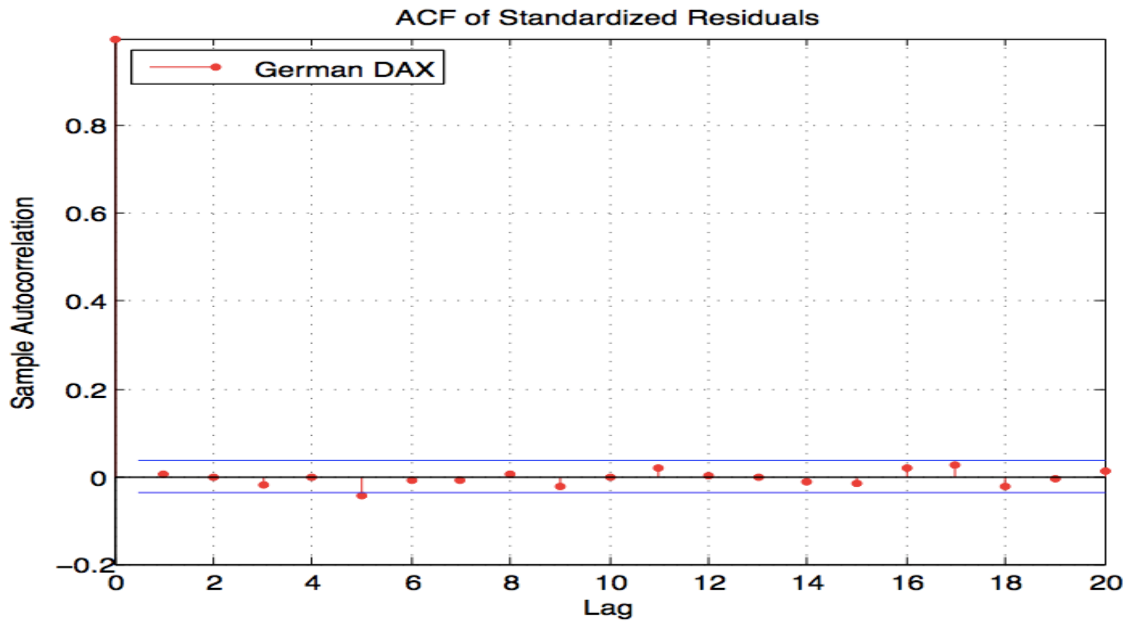

Figure 22: ACF of standardized innovation of German DAX index. Source: Yahoo! Finance.

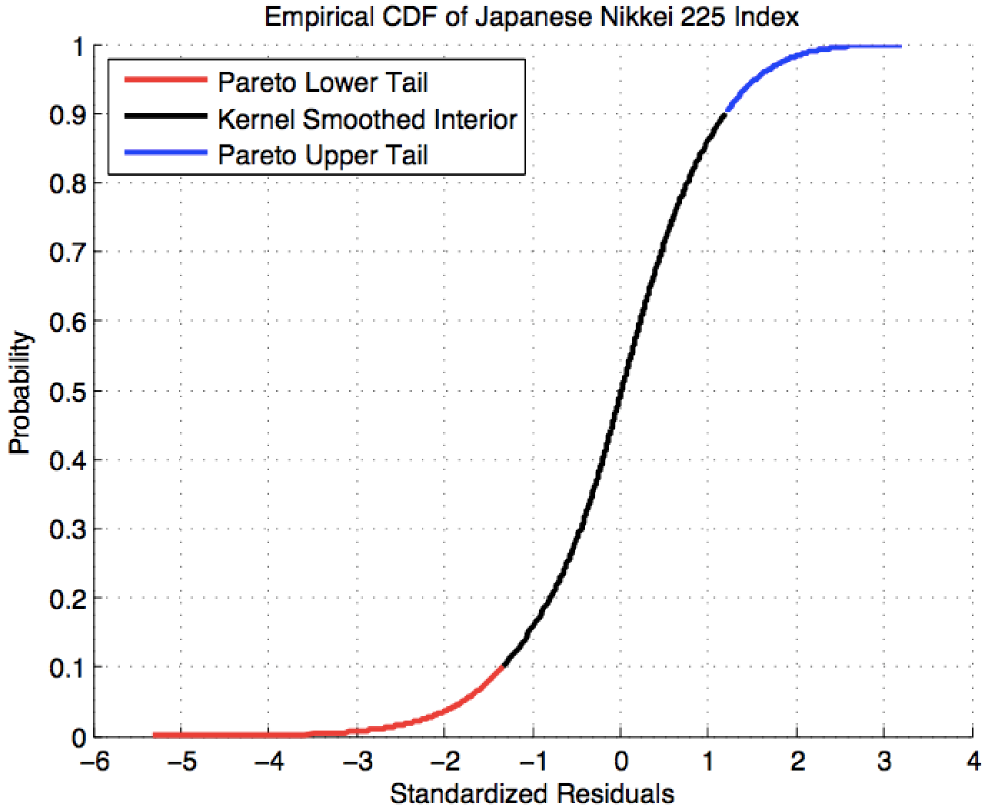

Figure 23: Empirical CDF of Nikkei index. Source: Yahoo! Finance.

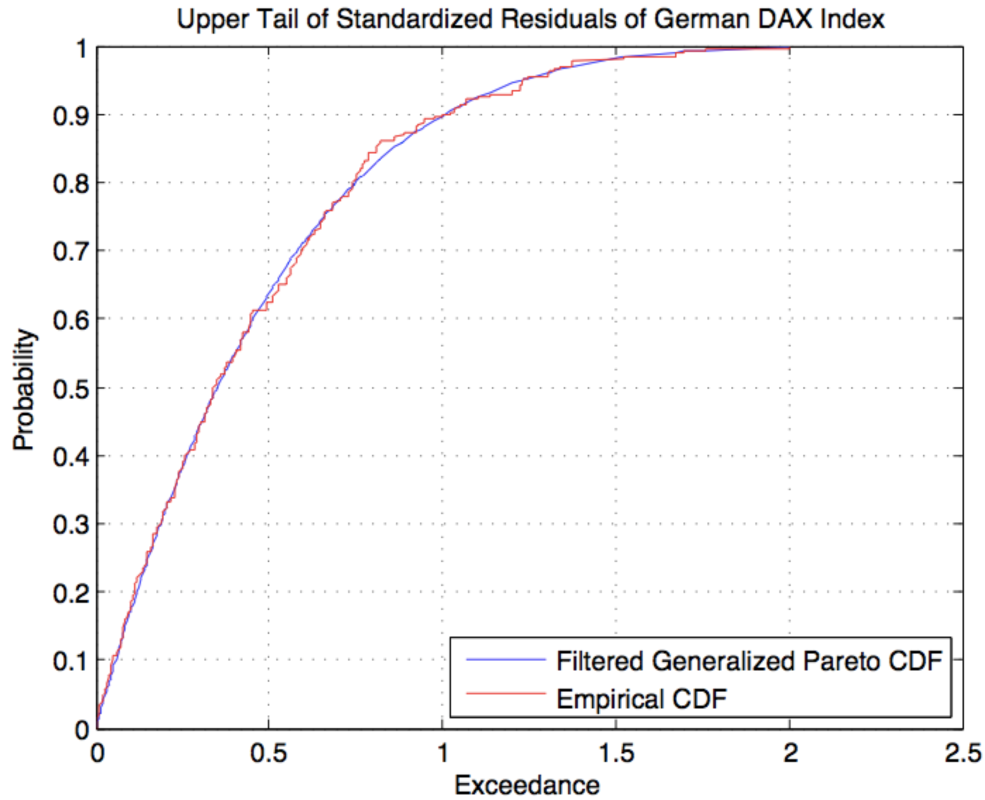

Figure 24: Filtered Generalized Pareto CDF vs empirical CDF of DAX index. Source: Yahoo! Finance.

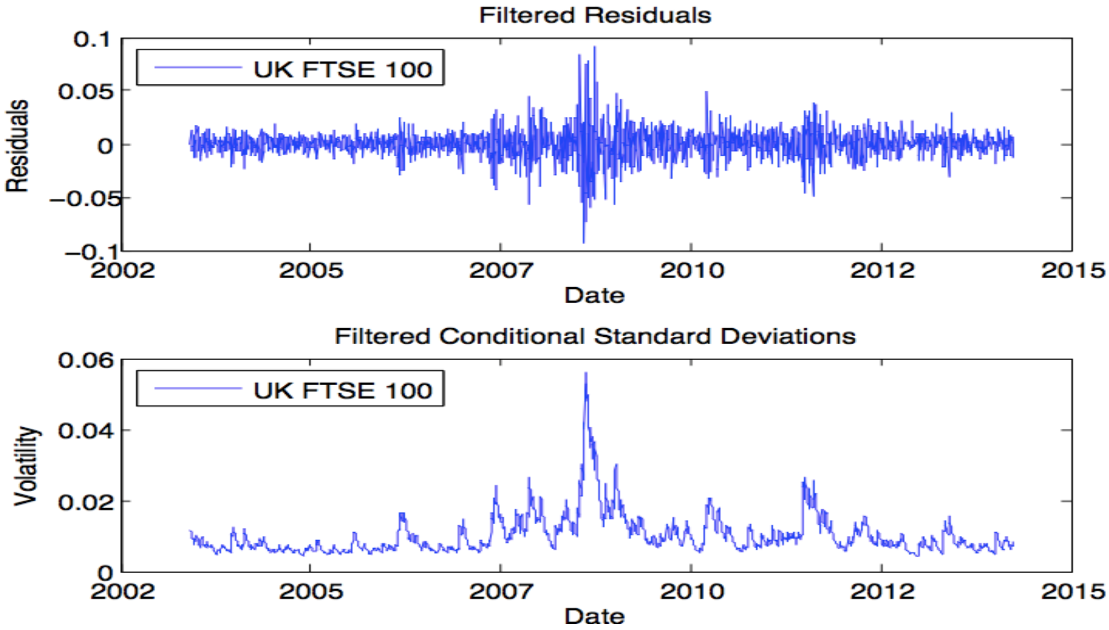

Figure 25: Plot of filtered innovations and filtered conditional standard deviation of UK FTSE 100 index. Source: Yahoo! Finance.

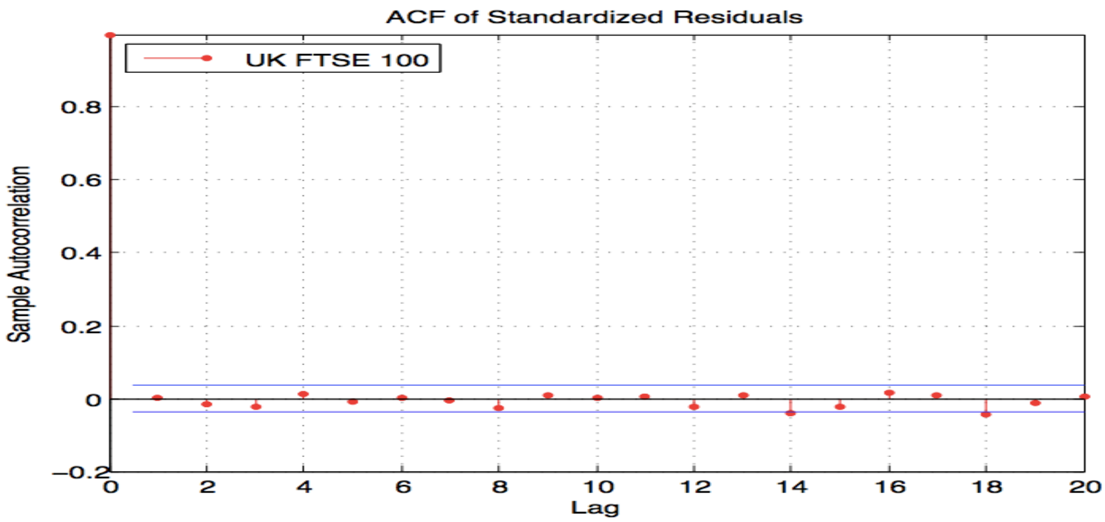

Figure 26: ACF of standardized innovations of UK FTSE 100 index. Source: Yahoo! Finance.

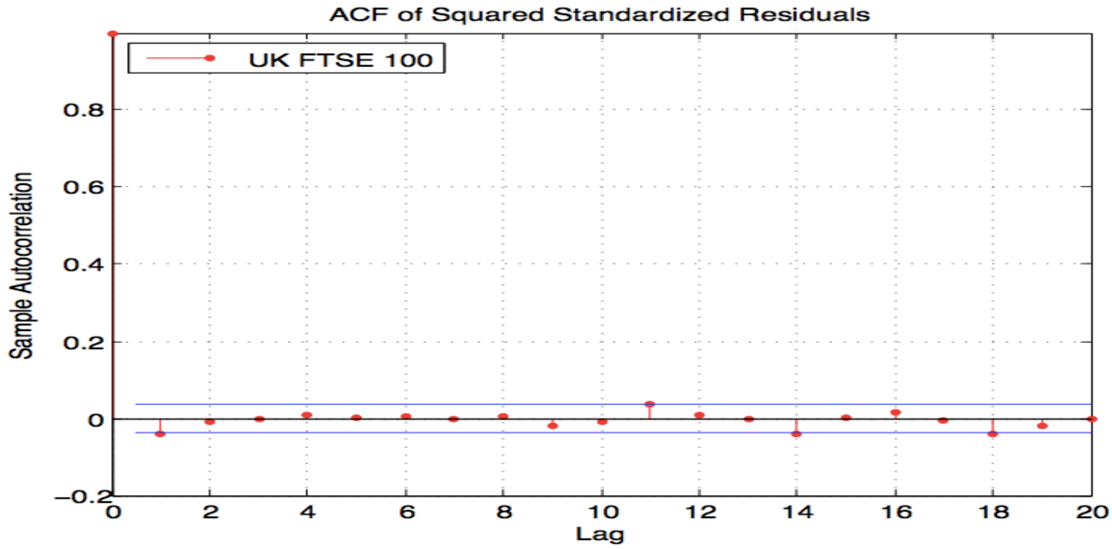

Figure 27: ACF of squared standardized innovations of UK FTSE 100 index. Source: Yahoo! Finance.

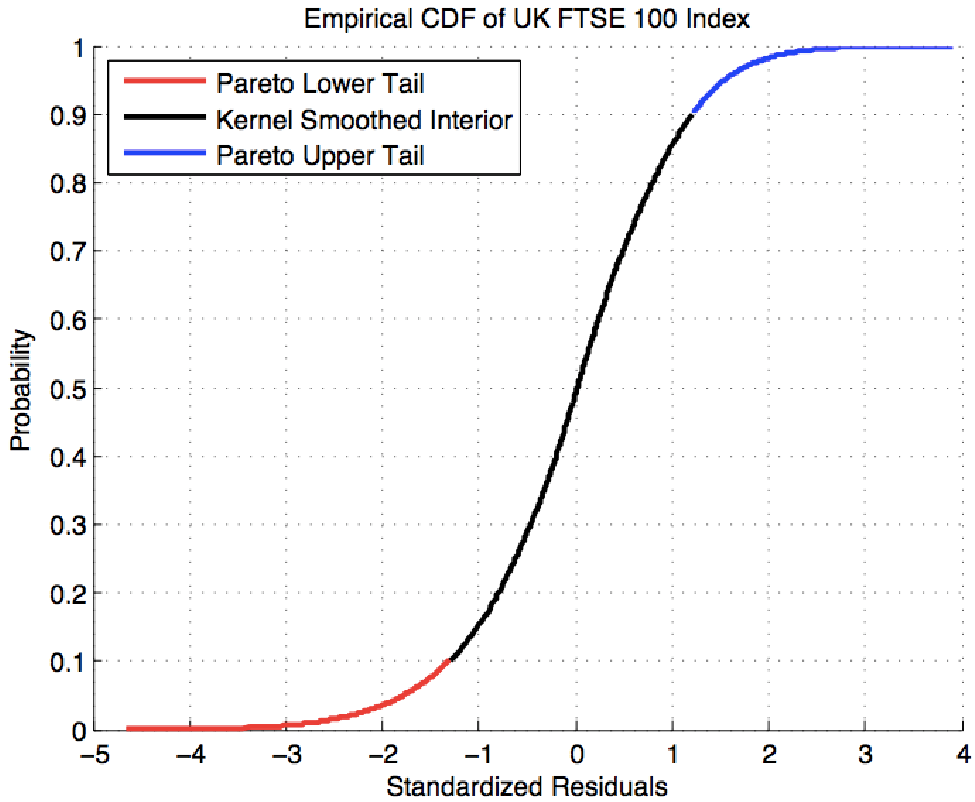

Figure 28: Empirical CDF of FTSE 100 index. Source: Yahoo! Finance.

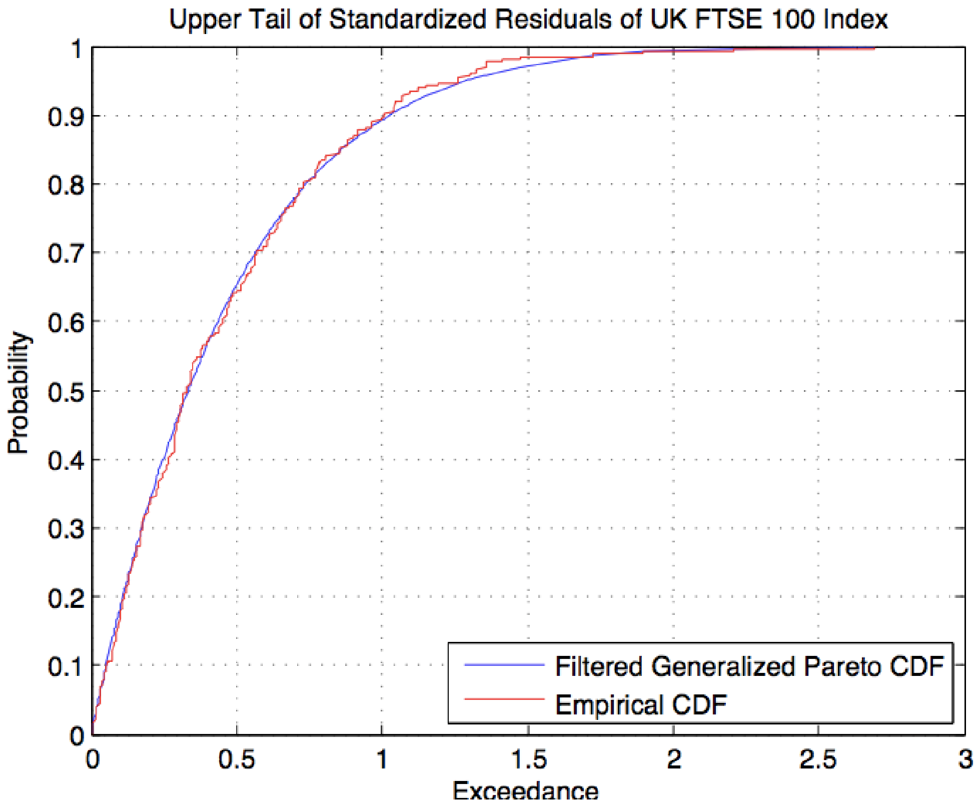

Figure 29: Filtered Generalized Pareto CDF vs. empirical CDF of FTSE 100 index. Source: Yahoo! Finance.

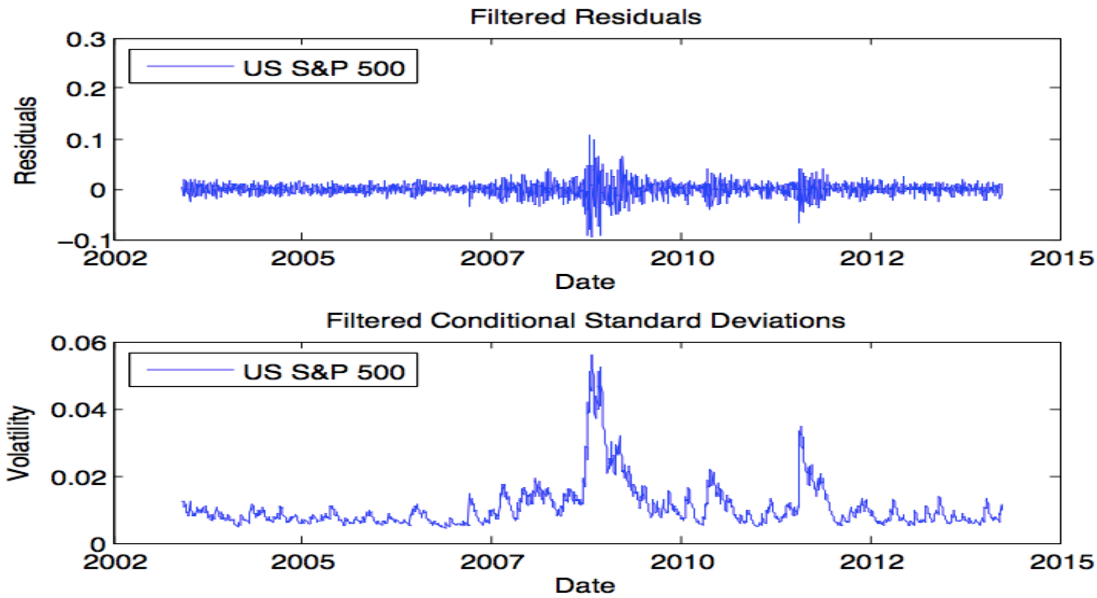

Figure 30: Plot of filtered residuals and filtered conditional standard deviations of US S&P 500 index. Source : Yahoo! Fiance.

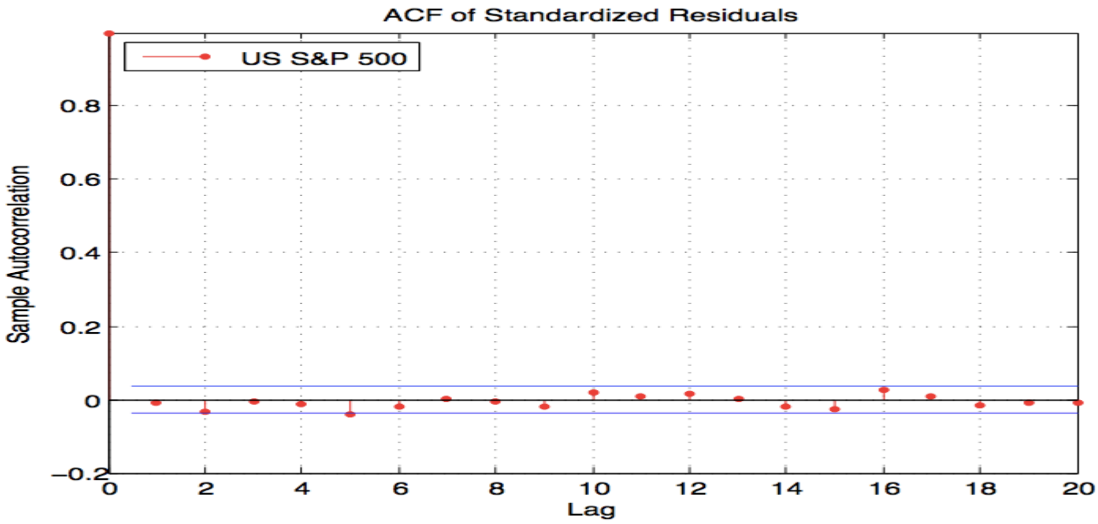

Figure 31: ACF of standardized innovations of US S&P 500 index. Source: Yahoo! Finance.

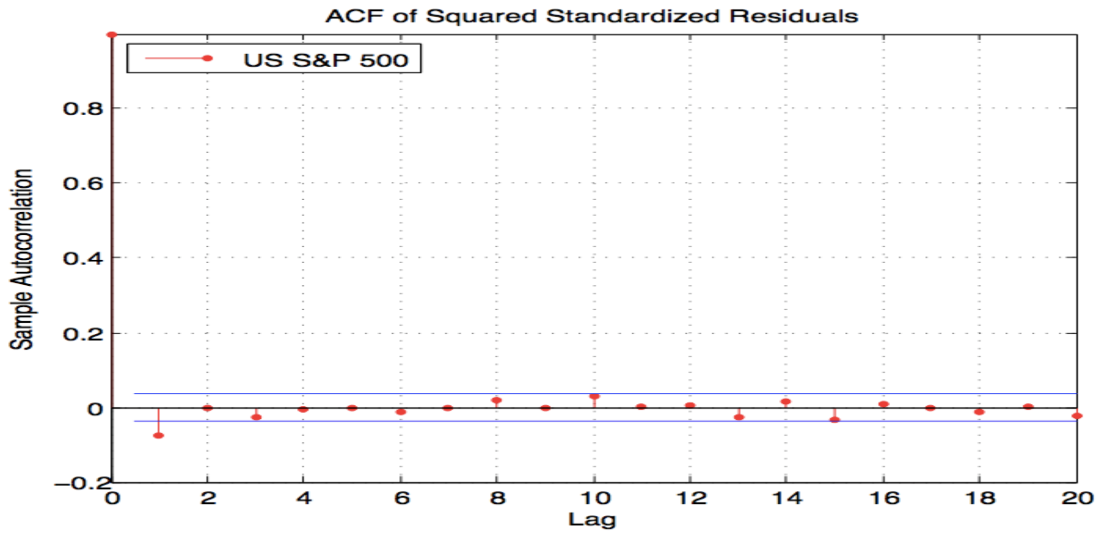

Figure 32: ACF of squared standardized innovations of US S&P 500 index. Source: Yahoo! Finance.

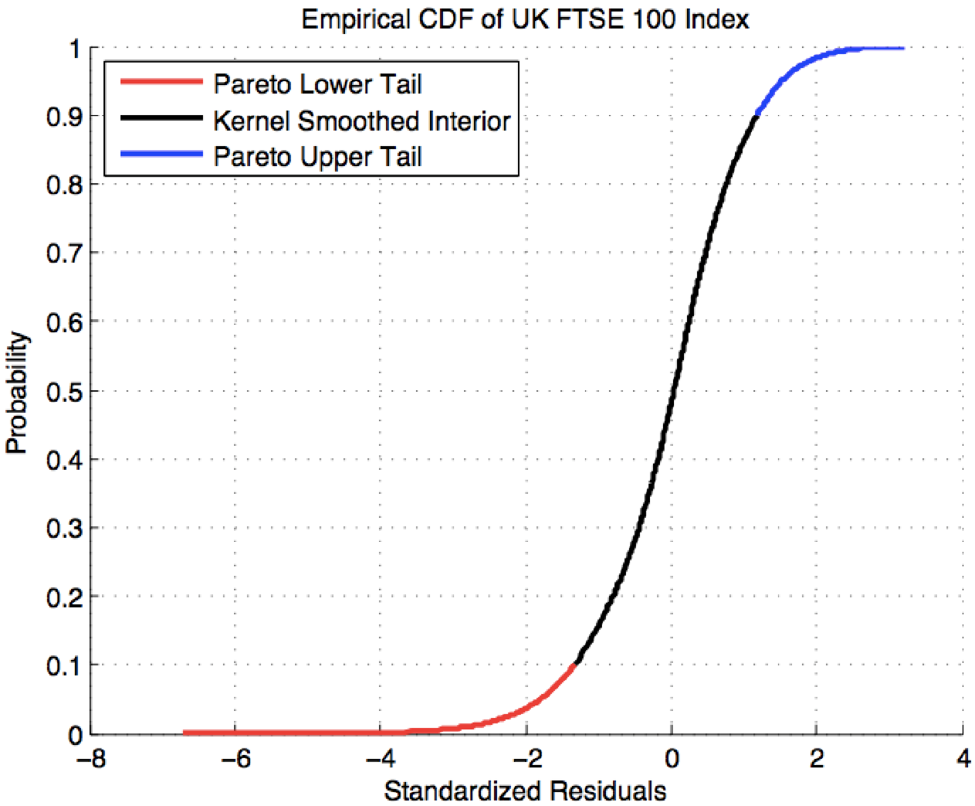

Figure 33: Empirical CDF of S&P 500 index. Source: Yahoo! Finance.

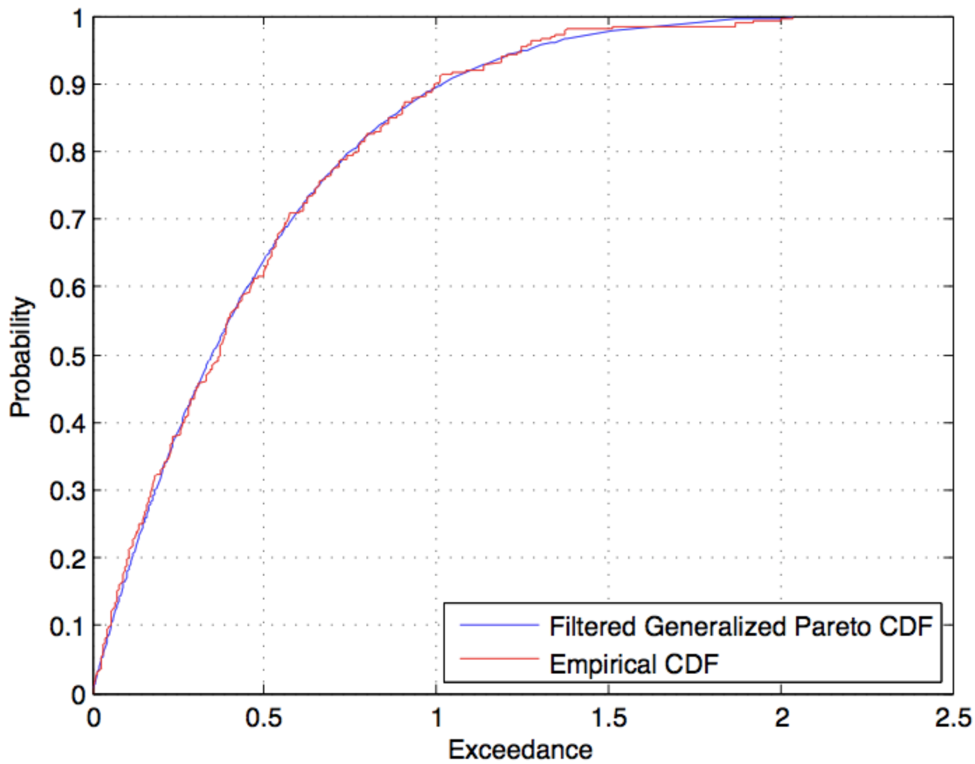

Figure 34: Filtered Generalized Pareto vs. empirical CDF of S&P 500 index. Source: Yahoo! Finance.

table 1
Advantage and disadvantages of VaR approaches.

		Advantages	Disadvantages
Parametric approach Makes a full parametric distributional and model form assumption. For example GARCH model with Gaussian errors		Its ease of implementation when a normal or Student-t distributions is assumed.	It ignores leptokurtosis and skewness when a normal distribution is assumed. Difficulties of implementation when a skewed distributions is assumed.
RiskMetrics a kind of parametric approach		Its ease of implementation can be calculated using a spreadsheet.	Its assumes normality of return ignoring fat tails, skewness, etc. this model lack non linear property which is a significant of the financial return.
Non Parametric approach (HS) Minimal assumption made about the error distribution, nor the exact form of the dynamic specifications		Not making strong assumptions about the distribution of the returns portfolio, they can accommodate wide tails, skewness and any other non-normal features. It is very easy to implement.	Its results are completely dependent on the data. It is sometimes slow to reflect major events. It only allows us to estimate VaR at discrete confidence intervals determined by the size of our data set.
Semi-Parametric approach some assumptions are made, either about the error distribution, its extremes, or the model dynamics	***Monte Carlo***	The large number of scenarios generated provide a more reliable and comprehensive measure of risk than analytical method. It captures convexity of non linear instruments and changes in volatility and time.	Its reliance on the stochastic process specified or historical data selected to generate estimations of the final value of the portfolio and hence of the VaR. It involves considerable computational expenses.
	CaViaR	It makes no specific distributional assumption on the return of the asset. It capture non linear characteristics of the financial returns.	Difficulties of implementation.
	FHS	This approach retains the non parametric advantage and at the same time addresses some of HS's inherent problems, i.e. FHS take volatility background into account.	Its results slightly dependent on the data set.
	EVT	Capture kurtosis and changes in volatility (conditional EVT).	It depends on the extreme return distribution assumption. Its results depend on the extreme data set.

Source: Author