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FVA MODELLING AND NETTING ARBITRAGE

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Introduction

After the Lehman Brothers default and the Euro Crisis, funding became a major issue in the industry. The fear of default was a major concern. CVA as the collateralisation of derivatives became a standard. In this stressed context, liquidity and funding turned into a huge management issue for market participants. Funding risk, which used to be the concern only of the treasury, was pushed to the trading desk. The industry has not yet clarified a standard practice regarding funding risk. The adding of this as a charge has been the subject of intense debate. Two main points of view have emerged: one that champions the inelastic assumption (fixed funding rate), and another that does the same for the elastic assumption (funding rate adjusted immediately after each transaction).

The definition of FVA is controversial, and not yet clarified. The literature is not well supplied with discourse about the actual mathematical definition of FVA. We can refer to [11], [12] and [13], but we prefer to define the term ourselves, and to present other points of view. To this effect, we present a valuation hedging methodology derived from various cases.

More than CVA, FVA relates to hedging strategies. As Kamtchueng [4], Piterbarg [7] and Burgard et al. [9] note, the hedging portfolio can also provide some cash. We can end up with different sources of funding. Piterbarg [7] has developed a PDE that takes into account the derivative seller's funding spread. Its results are based on the use of risky asset as collateral, which allows for cheaper funding than that available from the treasury.

The nature of the hedging security matters. Indeed, some of them can provide cash flow, and should therefore be considered also as a source of funding. But in practice, its is more a netting effect than a proper funding source. Indeed, if our hedging securities provide us with cash flows, our hedge sellers should charge us for the funding costs they generate.

Our main results are the following: First, we advance our default risk definition of FVA. Then we describe the impact of hedging strategy on the value and computation of FVA. FVA is a function of our funding spread, and in some cases, a function of hedge-seller funding.

Notation

$\mathbf{D}$ set of derivatives
$\mathbf{D^+}$ set of positive derivative
$\mathbf{D_C^+}$ set of positive derivative which can be used as collateral
$\mathbf{D^-}$ set of negative derivative
$\mathbf{r}$ is the risk free rate (supposed OIS rate)
$\mathbf{r^L}$ is the Libor rate
$\mathbf{s_A^f}$ funding spread of the entity A
$\mathbf{r_A^f = s_A^f + r^L}$ funding rate of the entity A
$\mathbf{r_A^f = x_A^f + r}$ funding rate of the entity A
$\boldsymbol{r^R}$ repo rate

$\boldsymbol{D^f(t_0,T) = E^Q\left[e^{\int_{t_0}^T r_s^f ds}\right]}$

$\boldsymbol{D\ \ (t_0,T) = E^Q\left[e^{\int_{t_0}^T r_s\ ds}\right]}$

$\boldsymbol{FVA^{(A\to B)}(P)}$ the FVA charged by A to B concerning the contingent claim P.

$\boldsymbol{P^C}$ it is the collateralized contingent claim $\boldsymbol{P}$

$\boldsymbol{\widehat{P}}$ it is no actualised premium related to $\boldsymbol{P}$

$\boldsymbol{\Psi^+}$ it is the positive value of the payoff $\boldsymbol{\Psi}$

$\boldsymbol{\Psi^-}$ it is the positive value of the payoff $\boldsymbol{-\Psi}$

$\boldsymbol{C_{t_i}(t_j)}$ value of the collateral at time ti of the collateral posted at time tj

Definition of FVA

There is no well established definition of FVA. The scope of its adjustment is itself the source of debate (see [14], [15], [16] and [17]). However, we can agree on this form of working definition in the context of a collateralised portfolio $\boldsymbol{\pi_t^C}$:

$$\boldsymbol{FVA^{\pi^C}(t_0,T) = E^Q\left[\int_{t_0}^T e^{-\int_{t_0}^t r_s\ ds}\,\pi_t^{C-} x_t^f \alpha_t dt\right]}$$

We do not want an explicit $\boldsymbol{\alpha_t}$ for the different points of view available in the literature. Some authors consider FVA a default risk-free measure. They propose this form:

$$\boldsymbol{x_t^f = r_t^L - r_t}$$

We do not support this definition. Our FVA definition includes liquidity funding risk (specific to the seller). This funding charge makes sense, and avoids the DVA concern.

Remark: Some authors consider symmetric FVA. Others prefer a credit risk component with $\boldsymbol{\alpha_t} = 1_{\{\tau>t,\tau_c>t\}}$ as the joint survival distribution. We shall consider two cases:-one with CSA, and another without-CSA.

With CSA

We do not want to be focus on the collateral modelling. In a case of perfect collateralization, we have an immediate readjustment of the collateral:

$$\boldsymbol{P_t^C \coloneqq P_t - C_t\ (t)}$$

We could also consider $\boldsymbol{\delta_{t_i}^{P^C}(t_j)}$ the exposure resulting from the imperfect collateral assumption:

$$\boldsymbol{\delta_t^{\pi^C}(t^-) \coloneqq \pi_t - \pi_{t^-} + C_t\ (t^-) - C_t^-(t^-)}$$

Basically, the not-perfect-collateralisation exposes the seller (or the buyer) to the change of portfolio _ market value, and to change of risky collateral value. In addition, regarding the unknown market risk, the seller and buyer are agreed via the CSA terms to allow a certain amount to be put at risk:

$$FVA^{\pi^C}(t_0,T) = E^Q\left[\int_{t_0}^{T} e^{-\int_{t_0}^{t} r_s\, ds}\, \delta_t^{P^C}(t^-)^- x_t^f \alpha_t dt\right]$$

By considering rebalancing dates $(t_i)_{1\leq i\leq N^C}$, we have:

$$FVA^{\pi^C}(t_0,T) = E^Q\left[\sum_{i=1}^{N^C} e^{-\int_{t_0}^{t_i} r_s\, ds} \delta_{t_i}^{P^C}\left(t_{i-1}\right)^- x_{t_i}^f \alpha_{t_i}\eta_i\right]$$

with η_i the rebalancing period or margin period.

Without CSA

Remark: the definition of FVA depends on the relationship between the trading desk and the treasury. The trading desk can decide to hedge its funding in the option market if its funding exposure is liquid enough. For liquidity reasons, we have to distinguish the risk-neutral value of the funding risk and its market value.

European Payoff

Considering a European option $\pi_t \in D$ maturing at T without any CSA:

$$\pi_{t_0}(T) = E\left[e^{-\int_{t_0}^{T} r_s\, ds}\Psi_T\right]$$

FVA can be expressed as follows:

$$FVA^{\pi}\ (t_0,T) = E^Q\left[\left(e^{\int_{t_0}^{T} r_s^f ds} - e^{\int_{t_0}^{T} r_s\, ds}\right) e^{-\int_{t_0}^{T} r_s\, ds}\Psi_T^-\right]$$

We prefer to introduce two different proxies, one market related, the other computation related:

$$\overline{FVA}^{\pi}$$

Market Proxy

In practice, the trading desk could decide, regarding the liquidity of the funding-risk exposure, to cover it via the option market.
Remark: This methodology implies an increase of credit risk for the selling counterparty (see [10]), and is subject also to liquidity market risk.

$$\overline{FVA}^{\pi}(t_0,T) = \pi_{t_0}^{mkt}\left(D^f(t_0,T) - D\ \ (t_0,T)\right)$$

The contingent claim seller decides to take a position collateralised on the funding exposure liquid market, and to finance his long position with a loan from the treasury, until maturity. There–is still some residuals risk, the funding resulting of the collateral call and credit risk being more or less negligible (depending on the counterparty's default probability and the exposure variation during the margin call).

$$FVA_{\perp}^{\pi}$$

Independent Proxy

To facilitate the computation of FVA, a common practice is to consider the independence of the process x_t^f t and the payoff Ψ_T :

$$FVA_{\perp}^{\pi}(t_0, T) = E^Q[\Psi_T^-]E^Q\left[\left(e^{\int_{t_0}^{T} x_s^f ds} - 1\right)\right]$$

Remark: If $\overline{FVA}^{\pi}$ and $FVA_{\perp}^{\pi}$ are similar, we have to mention the singularity of the market proxy. Indeed, we evaluate our FVA by our market-observation of our funding exposure. Therefore we are subject to other risks, such as liquidity market risk and credit risk. Secondly, in case of uncollateralised transaction, the hedging seller is entitled to charge us for his funding risk. Our derivation can be generalised easily to a multi-cash-flows-exchange-date contingent claim.

Perfectly Hedged without Cash Position

As a perfectly hedged portfolio, the cash flow that we owe to our buyer is replicated by the hedging portfolio. Therefore there is no apparent funding issue coming from the self-financing property of the hedging portfolio. (We assume the involvement no dynamic cash position.)

As an example, we can consider the statically replicable derivative (a subset of the perfectly replicable derivative, with no dynamic cash position involved in the hedging strategy). In this simple case, it is clear, as it shown in Appendix 7.1, that we are subject to the funding charges pursuant to our decomposed hedge portfolio:

$$FVA^{(A\to B)}(P) = FVA^{(Y\to A)}(P^+) - FVA^{(A\to X)}(P^-)$$

To avoid netting arbitrage, the seller derivative has to take into account the FVA of his hedging portfolio. Therefore, our FVA should be a function of s_A^f and s_Y^f . One major result is that our FVA can be sensitive to other funding spreads.

Perfect Hedge with Cash position

In this context, a hedging strategy with a dynamic cash position, we need to consider a founding strategy. The hedging strategy can be self-financing, with negative cash position. Therefore we need to find a way to fund this negative position.

The classical way is to borrow money from our Treasury at rate r^f . An option would be to use a part of our long position on risky asset as collateral, which is a cheaper way of funding ourselves. So, considering the Repo Market, we are able to diversify our funding sources, as shown in Appendix 7.3.

We have to make some comments regarding the way we are willing to handle our risky asset position (see Appendix 7.2):

First of all, this is a choice. The trading desk can choose to lend the asset to fund itself at the rate r_R. This choice is not only a dependant of the Repo Market but also a utility of the desk regarding the other frictions implied by this trade –frictions such as repo management, credit-risk limit, etc.

As noted by Kamtchueng in [10], there are many ways of using our security or contingent claim as collateral. Some options can be added to the trade regarding the transfer

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of dividends or ownership of coupons. As the result, the repo rate will be impacted according to ownership as established by the deal.

The controversy about FVA applies at this level in the sense that the security or contingent claim can be more valuable in our Equity (assets of the firm) than outside it (in the Repo Market). Indeed, on the elasticity assumption, our funding rate r_A^f will be modified automatically. On the other hand, inelasticity in the Repo Market can value the collateral quality of the asset at the rate r^R.

We can note that if it were always beneficial to add the security or contingent claim to Equity, the Repo Market would be useless, in the sense that the security would have more impact on our funding rate.

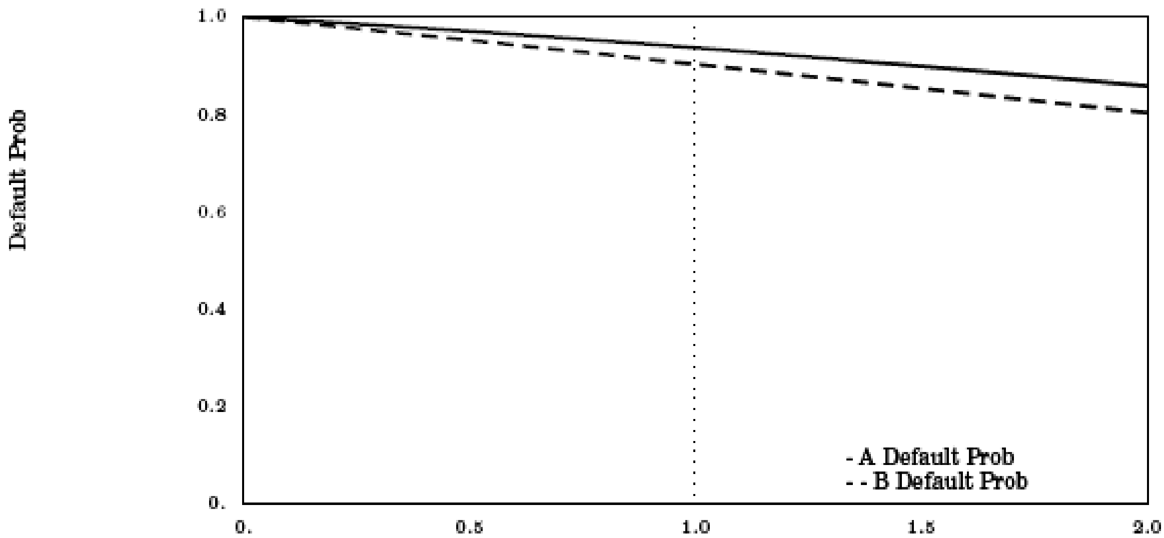
Application to a Synthetic Forward

A synthetic forward can be replicated by a static position on a long $Call_{t_0}(T,K)$ and short $Put_{t_0}(T,K)$:

- $FVA^{FwdS}\ (t_0,T) = E^Q\left[\left(e^{\int_{t_0}^T x_s^f ds} - 1\right)\Psi_T^{Put}\right]$
- $\overline{FVA}^{FwdS}(t_0,T) = Put_{t_0}^{mkt}\left(D^f(t_0,T) - D\ \ (t_0,T)\right)$
- $FVA_{\perp}^{FwdS}(t_0,T) = \widehat{Put}_{t_0}E^Q\left[\left(e^{\int_{t_0}^T x_s^f ds} - 1\right)\right]$

We have seller A, who has taken a long position on a Call with B and short one on a Put to X, having ventured a static hedging on short fwds. To perform our numerical test, we need to identify the funding spreads of A and B (see Figure 1).

Figure 1: Default Probability Term Structure



FVA was discussed in terms of its definition and relevance as pricing adjustment. We implemented the following FVA methodologies:

To establish the FVA netting arbitrage, we consider the standard method.

Table 1: FVA for Synthetic Fwd without CSA, strike=120, Maturity 2Y

Adjustment Type	$\overline{FVA}(P)$	$FVA(P)$	$FVA(P) - VaR$

A- Seller	1.9107	5.9404	24.808
B- Seller	3.3827	8.8978	35.527
A- Buyer	5.7194	12.845	119.16
B- Buyer	10.1256	12.135	180.45

$\overline{FVA}(P)$is the market-based proxy defined in 3.2.1.
$FVA(P)$ is abstracted via a Monte Carlo, and is based on the definition advanced at 3.2.1.
$FVA(P) - VaR$ is a percentile of the above Monte Carlo methodology (99% for our tests):

Using the result shown in Table 1, we obtain the following result:

$$FVA^{(A\to X)}(FwdS) = FVA^{(B\to A)}(Call) - FVA^{(A\to C)}(Put)$$

$$FVA^{(A\to X)}(FwdS) = 19.135 - 5.940 = 13.195$$

To hedge his position statically, the seller is sensitive to an unbounded funding exposure, the aggregated funding cost of 13.195, whereas the standard proxy is 5.490. This result will be a major issue in the negotiating process between the seller and the buyer. In our example, the seller is exposed to a bounded funding exposure. That is not the case for the buyer. The aggregate funding cost is a dependant of the identity of our hedge-seller B, and of the way he will charge A for funding costs.

Remark: Kamtchueng [10] has established a PDE for the CVA premium that takes into account different funding strategies. This is another example of the funding implication of the New Pricing Theory.

Conclusion

We have shown in this paper that FVA is very sensitive to our hedging strategies. Indeed, it was established that the choice regarding our potential funding sources can impact our funding valuation adjustment in many ways:

Even in case of the perfect self-financing strategy, we can be subject to the FVA cost produced by our hedging portfolio, and therefore to other funding spreads.

The trading desk decides whether to use the liquidity of our hedging risky assets as collateral.

As it has been proved with CVA in [4],[5] and [6], the industry should take more note of hedging strategy before computing adjustments based on mathematical risk measures. Communication between traders and quants is essential in the achievement of a relevant quantification of risks.

The definition of FVA implies an asymmetric fair value that will impact the entire market business. The trading desks have to find a consensus on what can be a benefit or a cost in derivative. A conceptual remark has to be made about the measure computes FVA: If it is treated as a cost, there is no reason to consider the risk-neutral measure. This subjectivity is another source of debate that will be analysed in another study.

The pricing status of the FVA is one of the subjects of The ‘Default’ Fear Pricing Theory – CVA and LVA [14].

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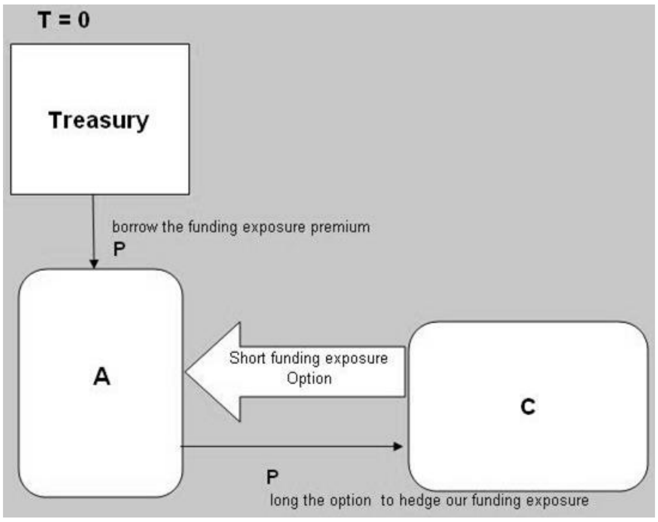
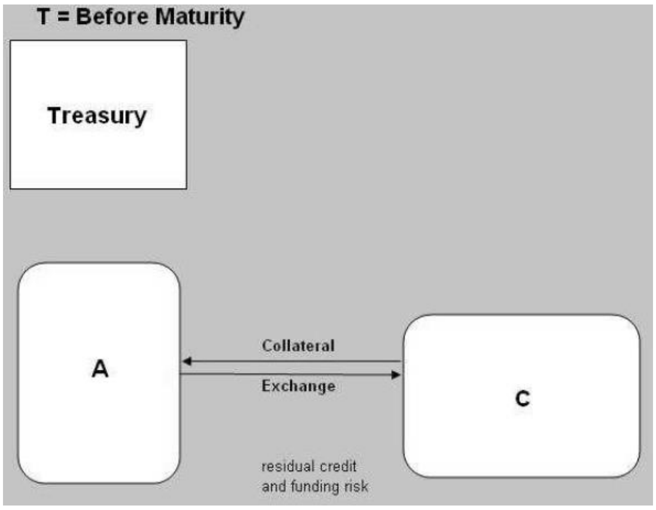
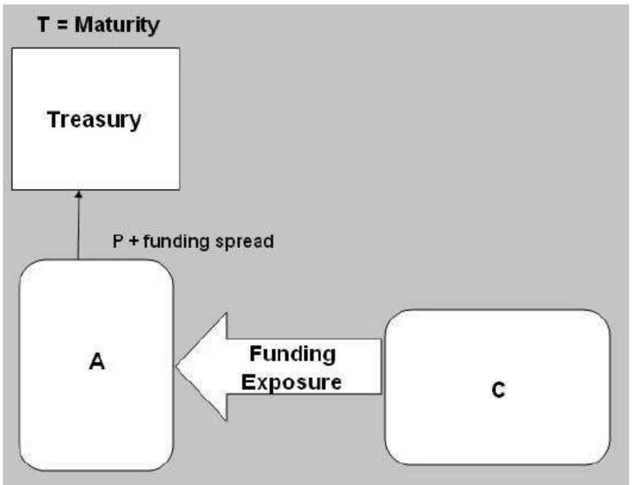
Appendix

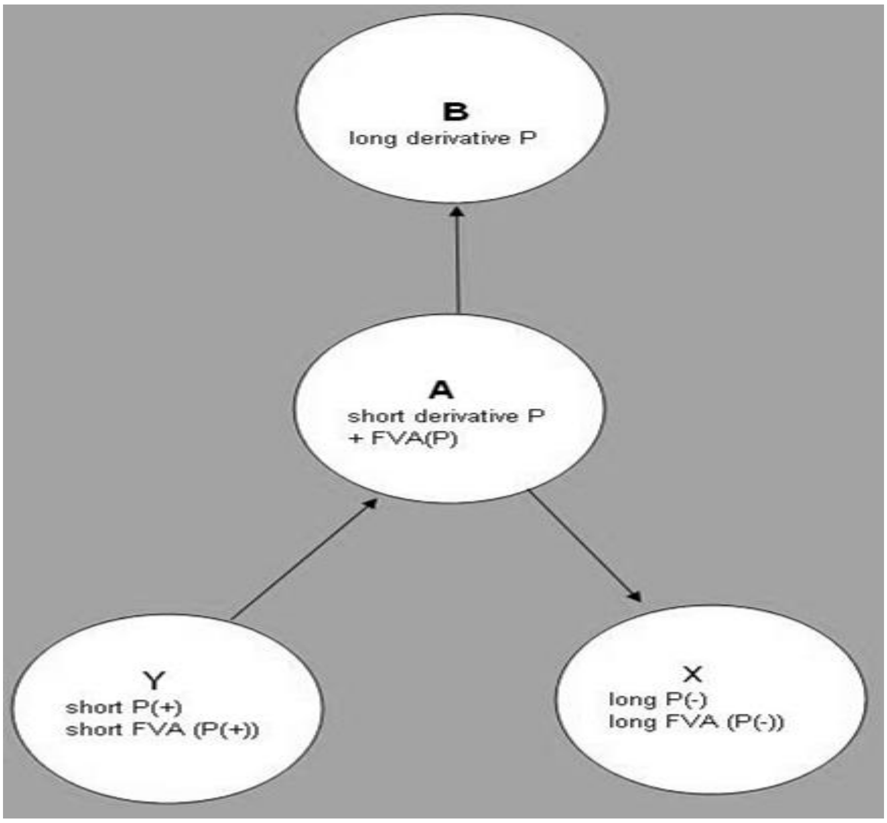
FVA Market Proxy Strategy



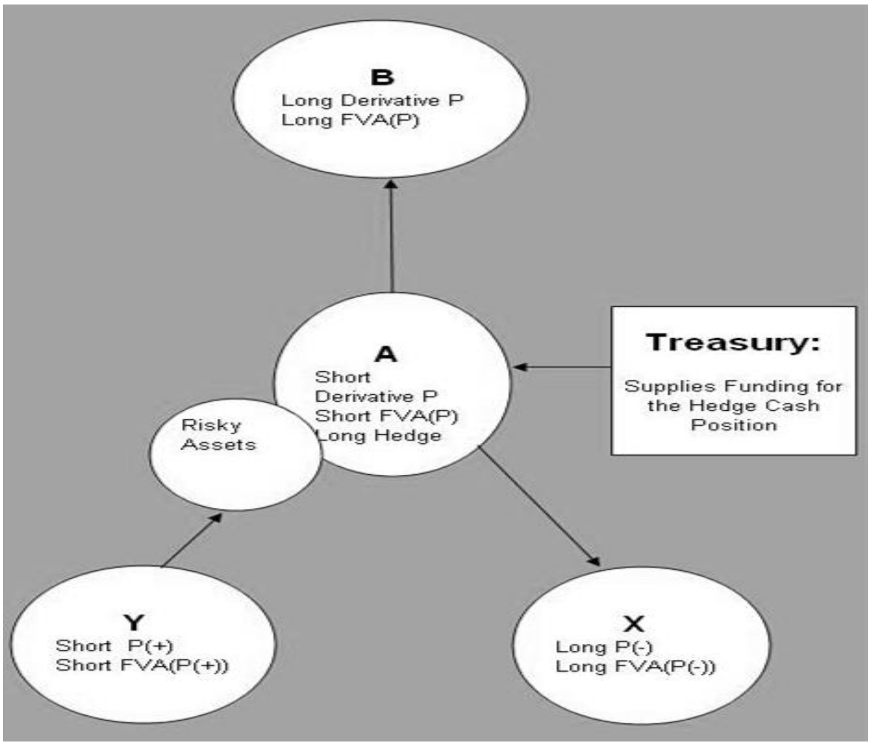




FVA Perfect Hedge without Cash



FVA Perfect Hedge with Cash Position with Risky Asset



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FVA Perfect Hedge with Cash Position